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EVALUATING THE INTEGRATION OF WORD PROBLEMS, WORLD EXPERIENCE, AND MATHEMATICAL KNOWLEDGE IN YOUNG CHILDREN

Linda Yuet Ling Kwan
Department of Psychology, The Education University of Hong Kong, Hong Kong
ylkwan@eduhk.hk

Abstract

This study arose from research conducted in a school where students aged seven to nine struggled to solve mathematical problems. The study's goal was to find out how children make sense of their problems. Students were given a few simple arithmetic problems and then individually interviewed to determine and comprehend the difficulties that the students were experiencing. The problems' stories involved a quantity being increased by or combined with another quantity to form a total. The quantities were small natural numbers that did not exceed 20. The findings revealed a number of problems with mathematics learning. The results were derived from how students understood the word problems, the relationship between the word problems and real-life experience, the relationship between real-life experience and mathematical knowledge, and the integration of word problems, world experience, and mathematical knowledge. How students work and verify their answers in order to better understand their thinking was observed. The usefulness of word problems in school can be realized only if students' understanding of a particular situation can be *elicited, enriched, or embellished with their experience before that experience can be re-examined in light of the theory that is applied to the real-life situation.*

Keywords

World Experience, Word Problems, Mathematical Knowledge, Real-Life Experience, Arithmetic Problems

1. Introduction

Teaching children to solve word problems is an important component of any mathematics curriculum (NCTM, 2000). When students are required to analyze and solve word problems from their own lives, they can broaden their problem-solving strategies. Students can learn that there can be more than one solution to a problem by solving word problems, and that what they learn from getting an answer wrong can help them figure out how to reach the correct solution (NCTM, 2000). Mathematics is said to be one of the gatekeepers to success in all areas of life. It is regarded as a key to problem solving rather than merely a subject. It is said that mathematics prepares students for practical life. Students can develop their skills, knowledge, analytical and logical thinking as they learn mathematics, all of which can help stimulate their curiosity and improve their problem-solving ability in so many areas of life. According to the National Council of Teachers of Mathematics, "problem-solving is an integral part of all mathematics learning." Being able to solve problems in everyday life and at work can provide significant benefits' (NCTM, 2000). When students are required to analyze and solve word problems from their own lives, they can broaden their problem-solving strategies. The current study sought to discover how students aged seven to nine years solved mathematics problems.

The study was designed to address the following questions: How did the children make sense of the problems? In particular I look at:

1. The relation between word problems and real-life experience,
2. The relation between real-life experience and mathematical knowledge; and
3. Bringing word problem, world experience and mathematical knowledge together

2. Literature Review

This section examines the literature on solving word problems in mathematics as well as the difficulties that children face when solving word problems. It also explains the study's conceptual framework in relation to the literature.

Teaching children how to solve word problems is a significant part of any mathematics curriculum (NCTM, 2000). It is also a fundamental life skill necessary for students to solve everyday real-world problems (Bottge and Hasselbring, 1993). The ability to solve word problems makes it clear to students just how useful mathematics is (NCTM, 2000) by asking them to provide solutions to everyday issues requiring statistics, algebra, probability and geometry. When students have to analyze and solve word problems which occur in their own lives, they can expand their problem-solving strategies. As students reflect on their strategies, when seeking solutions to word problems, they are encouraged to practise logical thinking. By solving word problems, students can see that there can be more than one solution to a problem and that, what they learn from getting an answer wrong can help them work out how to reach the correct solution (NCTM, 2000). Mathematics problem-solving is also an aid to the development of vocabulary and language – and not just in a student's receptive language (as they try to understand what a word problem means) but also in their expressive language when they have to explain their results and their thought processes both in writing and verbally (Cobb, et.al., 2003;).

It has been said that mathematics is one of the gatekeepers for success in all life fields. It is considered a key for solving problems – not just a subject. Mathematics, it is said, prepares students for practical life. As they learn mathematics, students can develop their skills, knowledge, analytical and logical thinking, all of which can help stimulate their curiosity and improve their problem-solving ability in so many fields of life. The National Council of Teachers of Mathematics puts it this way: 'problem-solving is an integral part of all mathematics learning. In everyday life and in the workplace, being able to solve problems can lead to great advantages' (NCTM, 2000, p. 52).

It has long been suggested that the teaching of mathematics should shift from the acquisition of procedural proficiency to the development of problem-solving skills (NCTM, 2000). Word problems are used to connect the mathematics the students are being taught to real-life situations they could encounter day-to-day. The idea of a word problem is to apply mathematical theory to a real-life situation. Such word problems have been a significant feature of a mathematics curriculum and are considered a way to develop students' general problem-solving abilities and promote a deeper understanding of how mathematics operates (Verschaffel, 2002; Yeni, E. M.,2015).

The structure of a given problem influences how a child solves a word problem. Word problems can be used as a basis for application and a basis for integrating the real world into mathematics education. They can provide practice with real-life problem situations, motivate students to understand the importance of mathematical concepts, and help students to develop their creative, critical and problem-solving abilities. Verschaffel (2002) described their goal as 'to bring reality into the mathematics classroom, to create occasions for learning and practicing the different aspects of applied problem-solving, without the practical ... inconveniences of direct contact with the real-world situation' (p. 64). Boaler (1994) added to this by offering three reasons for learning in context: to provide students with 'a familiar metaphor, to motivate and interest students', and to enhance the 'transfer of mathematical learning through a demonstration of the links between school mathematics examples and real-world problems' (p. 552).

Problem-solving in meaningful contexts, connections within and outside mathematics, and formal and informal reasoning and language underlie all areas in mathematics (Ardi, et. al., 2019)

3. Methodology

This section describes the methodology, the qualitative research methods used, the participants, and the instruments used. It discusses the procedure used, the use of interviews, the method of analysis, and the means of establishing credibility for my study.

The methodology was face-to-face interviews when the children were presented with three kinds of simple arithmetic problems (simple addition, missing addend and combination problems) and observe how they tackled them. There was no teaching or tutoring involved – it was simply observing with any interaction being limited to prompting, asking questions and answering. All of the interviews were recorded – by audio and/or by video. The problems' stories involved a quantity being increased by or combined with another quantity to form a total. The quantities were small natural numbers that did not exceed twenty. The study's goal was to find how children make sense of their problems. My study concentrates on the personal think-aloud interviews with 27 students to find out how difficulties arise when the children solve simple mathematics word problems.

The purpose of my study is to understand the difficulties of the children have in handling simple arithmetic word problems.

The focus is on problems which are of the most basic kinds with which the children, as their teachers said, would still have difficulties. The problems involve stories in which a quantity is increased by or combined with a certain quantity to form a total (instead of for example stories about handling comparison or differences between quantities, which are shown in the literature to be more difficult). The problem asked can be solved just by simple addition or subtraction. The quantities involved small natural numbers not exceeding 20.

3.1. Conceptual Framework of The Study

The study was to examine the difficulties children encountered in solving problems. I looked at the difficulties children encountered in reaching conceptual understanding and using procedures to help them do so and thereby solve a problem. I looked at the extent to which they could apply or gain conceptual knowledge by discerning common threads of variations within and across particular problems. I also considered carefully the variations in the students' understanding of each of those problems.

I also consider whether the children constantly make sense of what they do in their representation and operation of mathematics in relation to their world experience, or see the mathematics and their world experience as two totally separate things. The role of word problems is supposed to bring mathematics and the world together. Whether or how that can be achieved is fertile ground for others to research (Figure 1).

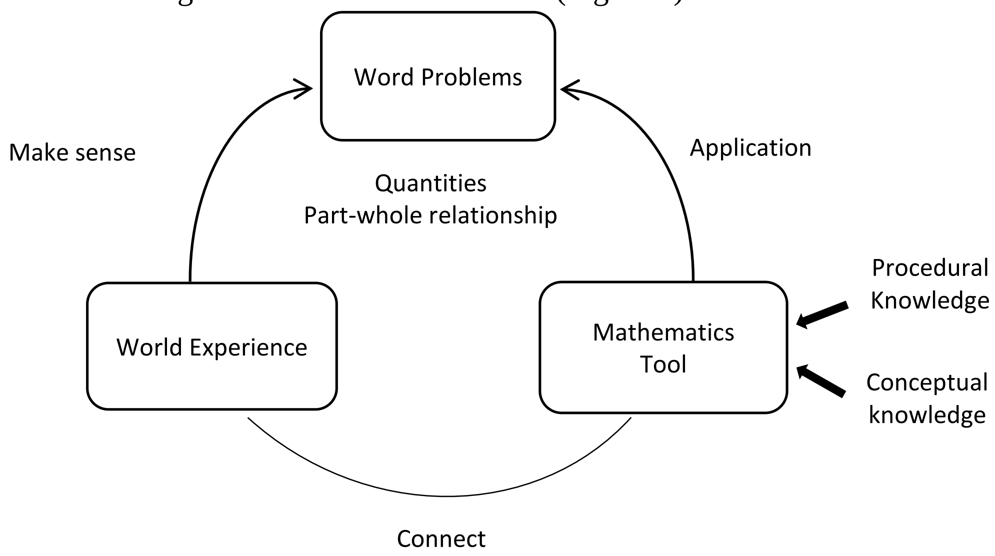

Figure 1: *Conceptual Framework*

(Source: Linda Yuet Ling Kwan's Own Illustration)

The process I followed was this: I set the students three kinds of problems - simple addition, missing addend, and guessing game. Then observed the processes they employed and the mathematics tools (i.e., both procedural and conceptual knowledge) they used in order to find the answer. Watching and listening to the children as they went about trying to solve a problem allowed me to observe the range of tools at their disposal and the extent to which they were able to use those tools effectively. If they found a procedure ineffective or they were uncertain whether they had got the right result, they may need to do some checking and adjustment. It also helped me identify whether the children had really made sense of the problem and whether they understood quantities and part-whole. Observing how many procedures the students had available to them, and how well they could use these procedures, helped me understand their difficulties (Wijaya, et. al., 2019).

If the child has not had analogous experiences in their day-to-day life, they may have difficulty in understanding the problem situation. On the other hand, if the child has had analogous experiences in their day-to-day life, then this may prompt a connection between that experience and their classroom understanding of mathematics. This experience might result in their coming to understand the relevant concept or prompt them to consider an alternative procedure.

I was interested to explore what world experience the children could bring to bear when attempting classroom mathematics problems. World experience is extremely important because children can see a problem in context. It helps build their web of understanding. Without analogous world experience children are left with attempting problem-solving simply by using the procedures they had learned in school - all theory, no practice. For a student to have a real understanding of the mathematics they need the combination of conceptual knowledge and a wide range of, and the ability to use, procedures and world experience of problems analogous to the theoretical situations they face in class. It is the linking of all these that enables a child to learn mathematics.

So, I was looking to see the extent to which children could connect the mathematics tools at their disposal with whatever world experience they had in order to apply these tools and experience to make sense of (and solve) word problems involving quantities and part-whole.

3.2. Interview and Video-Recording

One of the most important aspects of gathering data through interviews is the ability of the researcher to observe how the interviewee reacts and responds to questions. This enables the researcher to shift between the structured and unstructured aspects of the interview. For example, I might ask one of my pre-prepared questions and receive an answer which demanded further clarification which would then lead, immediately, into an unstructured discussion (‘Can you say why you did/thought that?’, ‘How did you decide that?’ etc.). The ability for a single researcher to observe, however, is rather restricted because the researcher has not only to stick to the structured questions but also use unstructured questions and there is little time to note (mentally or otherwise) how a student responded – not only verbally, but also in gestures, movements etc.

For this reason, I decided (having first obtained permission from the parents) to video record the second and third sets of interviews. Cohen et al., (2003) noted that, 'Observation methods are powerful tools for gaining insight into situations' (p315). I used two small and unobtrusive cameras to record myself and the interviewees so that, subsequently, I could pay particular attention to their body language and gestures, and the extent to which the students collaborated. At no time did the students take any notice of the cameras. They seem to be entirely relaxed throughout the whole process. The students could proceed entirely at their own pace and in a natural and informal way. This enabled me, subsequently, to review the video recordings several times and all this gave me invaluable insights into the students' procedural, conceptual, and strategic skills in solving mathematical word problems. In particular, I was able to glean information about what the students knew of mathematical concepts, how they could carry out mathematical operations, the procedures they employed when seeking a solution, and the sort of mistakes they tended to make.

I kept detailed notes of the process of the interviews. The elements of the interviews were also transcribed.

3.3. Thinking Aloud Method

I also encouraged the students to 'think-aloud' and explain their thinking whilst they were solving the problems. Getting them to verbalize their thought processes avoided the problem of relying simply on their written work, which discloses only their solution rather than the thought processes that went into reaching that solution.

Some students may be more fluent than others. Barrian is a good example of a student who 'thinks aloud'. She would do this without prompting and consistently. She did this by talking to herself – asking aloud for example, how, if she added X to Y would this come to Z? Barrian also counted aloud. The others, who were less fluent in thinking aloud, were prompted from time to time to resume thinking aloud, when they stopped doing so. But the researcher also needed to judge. When the child was too immersed in their thinking and could not say anything, the researcher might need to wait until slightly later to ask what they have been thinking.

It might be suggested that, asking children to 'think aloud' might mean that, just through verbalizing their thought processes, they thereby improved their skills, so that the problems they had before they approached the problem through thinking aloud were masked.

I acknowledge that as a possibility and, indeed, I observed this very phenomenon in the case of Barrian but there is a world of difference between just *watching* someone as they think quietly to themselves as they try to try and work out a problem and *listening* to how they go about thinking on the problem when they verbalize their thinking. I feel that, by asking them to think aloud I learnt a lot more about the issues they face than if I had not done so. Apart from that, I had already observed them in great detail attempting to solve problems in a way which suggested a distinct lack of both conceptual and procedural knowledge. So, I do accept that asking children to think aloud might result in an improvement in their skills, but it can be very revealing to the observer. It also gives a great deal of information – much more than if they didn't think aloud. In short, such disadvantages as exist when asking a child to verbalise the processes are far outweighed by the advantages. If we know how they reach their conclusion that this is far better than not knowing.

Quite apart from that, I used many data collection methods such as videotaping and others. The thinking aloud method was a complement to the other methods I employed. Moreover, I revisited the videotapes on many occasions and paid very close attention to everything the children did and spoke.

Asking a child to think aloud is far different from asking them, after the fact, to write down or explain their workings. The important phrase is, 'after the fact' because this is asked of the child only once they have 'solved' the problem, to show how they got there. Thinking aloud is very different – this happens before the fact. They may let you know some tentative ideas that do not work out afterwards. As Krieger and Martinez observe (2012) 'One of the methods now being used experimentally by medical educators to assess cognitive competence is the use of 'Think-aloud' interviewing methods employed in cognitive science studies of the reasoning process'. They note that, under this method, 'researchers ask subjects during the interview to verbalise their thoughts spontaneously as they emerge in attention'. This is very different from explaining thought processes after they have already gone through them.

It is the articulation of the thinking, as it happens, that makes thinking aloud so valuable. The student might suppose that 'A' is the way to solve the problem, but then thinks some more and decides that, in fact, 'B' is the correct way. 'After the fact' reasoning might concentrate only on 'B'.

Krieger and Martinez also say, 'we give students in a clinical program a hypothetical problem that is representative of work they have experienced in a clinical program, and record them as we ask him to talk it through. Our hypothesis is that by asking the students just to talk about a problem without a filter, we will understand, as well as possible, what they are thinking 'in practice.'

As Professor Scott Fruehwald (2011), in his commentary on Krieger and Martinez, 'Thinking-Aloud Techniques to Develop Problem-Solving Skills' states, 'I propose that not only are think-aloud techniques valuable in assessing student learning, they are a capstone technique in teaching students to solve legal problems. He quotes Krieger and Martinez: 'Course design must give students opportunities to develop the ability to reason in practice, and not simply to learn different expert techniques.'

Prof. Fruehwald comments that '… the process (i.e., thinking aloud) is just as important as the answer … It also helps students to develop the ability to deal with new types of problem, which is vital in practice.'

In my study, the thinking aloud did not take place in isolation from other activities. For example, as the children were thinking aloud about the problem, so they might also write things down – whether it was numbers or circles or whatever method they were using to help them solve the problem. So, I was able to observe the thinking aloud alongside the other things that they were doing.

3.4. Method of Analysis

The interviews were one-to-one. Among 27 students, four students were selected for a more detailed examination. I selected Pakiza who was typical of the majority of the 27 students in terms of the problems she exhibited in solving the word problems. I chose the other three, Addina, Zaima and Barrian, because, although they exhibited similar problems, there was something different about each of them.

Addina, for example, switched between two counting strategies. She would start with her fingers and, if this wasn't working for her, she moved to drawing circles on paper. She also displayed quite a lot of puzzling action during her problem solving, which called out for a deeper understanding. Zaima, alone amongst all the students I had observed, used gestures (she would 'put a number on her heart') as well as finger-counting.

I selected Barrian because she alone had shown a great improvement in her ability during the solution of the word problems I had posed. Moreover, Barrian showed that she was comfortable, in contrast to the other students, with larger numbers after her improvement. Furthermore, Barrian had an unusual way of counting. She used her fingers but she also counted by pointing with her finger at any object that might be on the table or, indeed my hands or shoulders.

I analyzed the performance of each student, one by one, and, in each case, analyzed each problem, one by one. I did this by reference to my field notes, any markings or workings the children had done on paper, and by viewing the video recordings. I took note of the difficulties the children had with the first problem and then when considering their performance in the second problem, observed whether they displayed those same difficulties and whether there were any additional problems. I then did the same for the third problem.

As regards the four children I had singled out for deeper analysis (Pakiza, Addina, Barrian and Zaima) I compared the performance of each of them as against the other three in respect of each of the three problems. I then took into account all the difficulties these four students had exhibited and examined whether some or all of these difficulties had been exhibited by the other 23 students.

Having analysed the data from these interviews, I then again carefully compared the performance of these four students with that of the rest of the group of 27 to see whether there were any common elements or any specific dissimilarities and to see if I could identify other particular difficulties the students had with understanding and handling the problems.

4. Findings and Analysis

This section presented my analysis and findings about the participants' work on the three mathematics problems separately.

4.1. The Difficulties That the Children Had in Their Process of Problem- Solving

Whatever the specific issues in respect of each kind of problem, the overriding difficulty for almost all of the students were the lack of conceptual knowledge as regards the part-whole relationship. This, more than anything, contributed to their difficulties.

4.1.1. Simple Addition

I observed limitations in the student's ability to subitize, to use counting strategies other than using fingers, failure to use grouping, and counting-on, and poor knowledge of number facts.

4.1.2. Missing Addend

The most striking difficulty was poor skills in translating a word problem into a number sentence. As I indicated earlier, there was strong evidence of the children having been moved into the use of number sentences before they were ready. I also observed limitations in the ability, accurately, to construct a number sentence. Even where a child accurately constructed a number sentence, most children encountered many difficulties in operating their number sentence. I also noted a lack of knowledge of alternative number sentence strategies (for example relying on an addition number sentence when a subtraction number sentence would have been better). I also noted a lack of knowledge of number facts.

4.1.3. Guessing Game

Lack of knowledge of number facts also featured in this type of problem. The failure to understand part-whole was particularly evident. Most students did not understand the idea of a unique answer and there was much evidence of a failure to understand the use of (and consequences of) variation in combinations of addends. Moreover, there is a distinct lack of the use of commutation (Hafid, et.al., 2016).

4.2. Differences In the Way of Making Sense of The Word Problems

The results were derived from the different ways in which the students understood the word problems, the relationship between the word problems and real-life experience, the relationship between real-life experience and mathematical knowledge, and the integration of word problems, world experience, and mathematical knowledge.

Word problems can help learners develop a deeper understanding and the ability to the mathematical knowledge learned and also helps them develop logical thinking in mathematics. Equally important, it also can connect the mathematical knowledge to the learners' real-life experience, so that they can make better sense of mathematics (and make mathematics also part of their common sense) and be convinced about the usefulness or meaningfulness of mathematics in the real world. However, the study reveals the reality that it is not easy to achieve these goals.

There are basically three things mentioned: word problems, mathematical knowledge, and world experience. However, the study reveals a number of questions that we need to look at in more detail if we are to build an interconnection among the three so that they would reinforce each other.

From the student's interviews, relations among mathematics knowledge, word problems and real-life experience are discovered below:

4.3. Relation Between Word Problem and The Real-Life Experience

Based on my findings for the Missing Addend Problems, we may need to be aware that the word problem we set might be seen differently by the students if it doesn't actually relate to their world experience. There can be two possibilities for this mismatch. First, the situation described in the problem may never have happened in the students' life. A mismatch of this kind is often easy to discover. You can ask whether the students have bought books or stationery, and they can tell you whether they have come across such a situation. Or you can ask them whether they have bought things themselves (or whether it is always done by their parents). The second issue, however, can be more obscure – even if they have come across such a situation, they might not have experienced it in the same way as we do. In setting up the situation, at the back of our mind, we have the mathematical knowledge, and we see the situation as matching our mathematical knowledge, so we may take it for granted that is the only way in which the situation would be seen.

In the Missing Addend Problem, the question says, 'You have 3 coins, and the book costs 7 coins. Do you have enough money? How many more coins do you need to buy the book?' Many teachers would assume this is a 'real world problem'. The students are likely to have real-life experiences of wanting to buy something but not having enough money. We may have this assumption about the learner:

Understanding that the item 'costs too much' and that the child 'does not have enough' means that they will have to pay attention to the gap – how much are they short? How long will they have to save up? How much will they have to ask their parents for?

The students are likely to realize that they do not have enough money. But they may go on thinking along different paths:

They might just give up and buy something cheaper or, perhaps, they might go home without buying anything; or they might realize that they still have more money in another pocket and so they take it out and count on and on, concentrating on the increase in total, until they reach the target amount or not - so they may abort the process, or they may just keep adding on more money until, hopefully, they get 7, without attending to the question how much more they actually add. In other words, they are attending to the changing sum, but not attending to (calculating) the difference.

World experience gives the child an awareness of quantities. But they might not see it in the same way as assumed in relation to the mathematic knowledge presented at school. In situations where the students just want to keep on adding money until it is enough, then the ordinal number actually can be quite helpful.

If the students have experienced a world situation in a way commensurate with the sort of frame of interpretation used by the school mathematics curriculum, they are much more ready to make sense of the word problem in the expected mathematical way. But, if there is a gap when a certain quantity (the missing addend) is actually not discerned and attended to in their handling of the world situation, but that quantity is assumed to be present and important in the mathematics problem (dressed up in that world situation), then the learner will find it much more difficult for to make sense of the problem in the intended mathematical way.

To summarize, we cannot just assume a problem that appears to the teacher to be a 'real-world problem of mathematics' must necessarily be seen in the same way in the eyes of the students. Problematizing this can help teachers to discover more possible obstacles which might have led to their students' difficulties in attending to solve the problem. Once we are sensitive to this, we can re-create the situation in a more open manner, so that our students can respond in ways they feel native, and from that understanding, we can begin to explore what can be done to bridge the different ways of seeing the situation. Teaching children in the context of the classroom without bridging the different ways of seeing can only be, I suggest, simply teaching the 'theory'. Perhaps the children 'get' the 'theory' but understood it only within a very limited context. If they are inexperienced in connecting the 'theory' in the way they learned in class to their real-life ways of handling the situations, such use of word problems may not actually help make mathematics more personally meaningful. Teachers might well wish to introduce some real-life examples to which the students can apply their classroom theory.

4.4. Relation Between Real-Life Experience and Mathematical Knowledge

In the study, it is also observed that, once a student categorizes the problem as either addition or subtraction, if the word problem does not make sense, they might still make use of the given numbers and work in a blinkered way following only the mathematical knowledge or procedures they have learned at school. This can result in the student losing connection with their real-world common-sense knowledge (and hence unable to check whether the result obtained is reasonable or not). This reminds us of the prime importance of making students see the corresponding relationship between mathematics and their real-world experience (their intuitive world).

Kant, the well-known philosopher, was actually also a mathematics educator. While he is famous for his rational analysis, it is also interesting to note his idea (Shabel, 2013) that numbers and basic computation operations must come from world experience. Children encounter any number of mathematics problems in their daily lives – only they don't see them as such. But they learn some mathematical concepts or ideas ('not enough', 'more', 'less', 'how many', 'how much'). This becomes part of their intuition knowledge. As Kant pointed out, a number represented numerically is utterly meaningless if the student knows only the concept but has not been shown how to understand it through an intuitive process. Kant poses the example '7+5=12' and says 'no matter how long I analyze my concept of such a possible sum (viz seven and five). I will still not find twelve in it.' The 'answer' could be stated as 'fifteen' or 'four', or any other number. As Kant puts it, one must go beyond these concepts (of seven and five) seeking assistance in the intuition that corresponds to them. His conclusion is that '7+5=12' can be established only by intuitive synthesis (Parsons,1969) and not by a conceptual or logical analysis (Anderson, 2004).

Adopting Kant reasoning, any numerical representation will only become meaningful when understood through intuitive learning. Children start learning mathematics by counting. They are taught to recognize spoken and written numbers, as well as symbolic numbers. But it doesn't mean that they understand them. They learn counting skills --- fingers, coins, lines, or whatever, which is essential for intuitive learning. Later Butterworth adds the importance of the intuitive grasping of quantity (subitizing) and grouping. They can learn by doing. Thus '7+5=12' is meaningless until first, the children understand that '7' means seven things, 'five' means five things, and '12' equals twelve things. Then using their counting and grouping knowledge and strategies on strokes to do the sum: 1111111 +11111=111111111111, and then the whole set of 111111111111 can be regrouped as a group of 1111111111 ('10') and 11 ('2').

However, Kant does not mean that the learning of mathematics should stop at the level of the learners' intuitive experience of the world. It must also be pointed out that the learning of mathematics does involve moving students beyond their original world experiences, to more logical possibilities in the realm of theoretical models.

Children also need to know that the resulting group must have ONLY one fixed quantity. It is not until the child understands that 5 can mean only five things and that '5+7 can only equal 12; that 12-7 can only equal 5 things that they really understand the part/whole relationship. But none of this, on its own, means that they understand part/whole relationship- namely the immutability of it all: 7+5 can only ever equal twelve. It is the basis on which the child knows that something yet directly known must be some fixed quantity and that it may be predictable from what is already known. This is what is fundamentally assumed when we talk about problem-solving in mathematics. In order that the student and the teacher can talk on the same 'meaning channel' about solving a problem, two conditions seem necessary: (1) The student needs to be helped by being presented with theoretical and hypothetical situations in the life world which are similar to mathematics problem; and (2) The student needs to be helped to see that the yet unknown quantity cannot be just random, namely, they have to check that some quantity can agree with the given known values in terms of some mathematical relationship, but that some cannot. This is about seeing the variation between whether a quantity can be correct or incorrect.

Understanding the inevitable consequence that 5+7 = 12 and 12 – 7 = 5 can be very difficult because it does not only depend on what real-life experience shows you. We also need those other possibilities that might initially appear possible and that, as a background to contrast against the actual instance we experience in front of us, to notice that there is in fact some further relationship that differentiates what is actually possible from what is not. With such reasoning, the mathematics teacher does have an important role to play, in prompting the students to see other related cases which are not immediately seeable in a situation. In this connection, some open-ended problems which are not a strictly real-life problems, such as the 'Guessing Game Problem' used in this study are also useful, and they can be brought about in connection to some problem situations of daily life, but extend it a bit beyond to a wider space of possible variations.

4.5. Bringing Word Problems, World Experience, And Mathematical Knowledge Together

We all make sense of problems in different ways and children certainly do when it comes to making sense of word number problems. For example, if they can relate the problem to a real-life situation, considering a comparative real-life situation alongside a word problem in class can better promote their understanding. But, not every child has wide real-life experience of handling money, so when faced with such word problems in class they simply don't see a connection with what has happened to them in their own life. They might simply try to relate the word problem to their existing mathematical knowledge, but fail to grasp the actual part-whole relationship in the problem situation. This can give rise to a failure to categorize the problem properly. It might well be a problem that can be solved using an addition number sentence but, the better way might be to use subtraction – but perhaps they can't see this. As the student cannot make sense of the problem, it is difficult for them to think about possible procedures and their selection. They may follow certain fixed rules (procedures) in a blinkered manner. Problems might also get complicated because the child does not understand the language of mathematics (the translation of words into symbols) including the grammar or operation of a number sentence. Losing the sense of the problem also means losing the means of checking whether their answers are reasonable or not (Moser, 2017).

In everyday mathematics lessons in school, teachers emphasize students' ability to use mathematical representation to solve word problems. Students do a repetition of mathematics exercises on one or two mathematics signs (e.g., + or -), without this leading student to an understanding of the process of making sense of the problems. Are the students able to draw diagrams with the problems? Will they understand that the missing amount of money cannot be greater than the cost of books? Have they discerned the uniqueness of a number? Will they notice variety of combinations of numbers? Will the teacher pay attention to these understandings from their students' point of view instead of expecting them to use their incomplete mathematics procedural knowledge?

Whatever we learn at school, the fundamental idea is to prepare us for real-life so, so when we are learning mathematics, whatever we learn at school should never be taught in a vacuum. Our education is to prepare us for life. When we learn the basics of mathematics it is to help us, to say the least, buy things, sell things, understand money, be financially literate, etc. Even though children might have little experience outside the school of handling or understanding money, most are likely to have at least some. It helps them with the concept of quantity – do they have enough money? Do they need more? How much does something cost? This means that when teaching mathematics at the primary level it is of the greatest importance that the children are able to understand the practical application, outside school, of what they learn in school. Of course, just as there are different levels of ability in any classroom, there will be different kinds of experiences among the children.

I would suggest that, in most families, there are countless situations in which children are faced (unwittingly) with mathematical concepts. 'Johnny has more candy than I do – it's not fair!). As indicated before, they may want to buy something with their own money but find that they do not have enough. Perhaps they don't have the mathematical knowledge from school to help them but, hopefully, children will have enough real-life experience at least to understand that they might not have enough money to buy what they want. This understanding can help them in school when, for example, faced with a missing addend problem. Perhaps they can consider their answer and ask themselves, 'does this seem right?' Maybe their real-life experience helps them realize that something is wrong – perhaps it is just intuition, but it will be intuition born of experience outside school.

Teachers need to understand this and simulate the real-world experience in the classroom by, for example, opening a 'shop' – giving the feeling of being in a supermarket and having to buy something; giving the children the opportunity to apply their mathematical knowledge in a real-world situation. A simple example would be to give the children 'money' in the form of counters or coins (or whatever), have items for sale (maybe they can draw or paint goods in art class), and then give them the chance to go to the shop and buy whatever items they desire. Perhaps pair children up and send them on a 'shopping expedition'.

Simulating real-life situations in the classroom can, I suggest, help children overcome problems of keeping track of the meaning of intermediate results in computation. Most of us know from our own experience that, when we are in a shop and we want to buy something, we count money out. Perhaps we put three coins in one hand and then we find the balance that we need in the other hand. This is a very common practice for children. It helps them understand grouping. It helps with counting. It helps with part-whole. The more real-life situations a teacher can introduce into the classroom when teaching mathematics, the better. After all, isn't that what teaching mathematics is all about at that level?

In this way, teachers might be able to compensate for any lack of real-world experience from which some children might suffer. In the case of my students, the strong impression I came away with was that they had very little if any real-world experience. This might well be down to their culture or simply the way that their family behaves but, in any event, in whatever school, I would strongly advocate combining real-world role-play with mathematical knowledge.

I would also suggest emphasizing the opportunities for students to create problems, suggest alternative solutions, and check the validity of solutions in the context of the simulation. In the study, it was noted that checking alternative solutions against real-world situation can help discern mathematical relations between the different quantities in the situation. However, from a teacher's perspective, my own experience is that students often prefer to present the teacher with the answer they have come up with and ask the teacher if they have got it right. That is not very effective for learning. Experienced teachers know that it is necessary to put the responsibility for checking on the student - and checking does not merely mean that they have carried out their procedures properly, but that they have understood (made sense of) the problem and put their answer back to that situation of the problem.

Some students in the study used the trial-and-error method but students can be helped to generate more effective variations. One example in the study was that, in the missing addend problem, a child checked that one answer was wrong, and then she randomly replaced it with another guess. So, we cannot assume that trying out, or even empirical testing out of tentative answers using concrete material must lead to the discernment of the underlying relationship. However, if the child was prompted to consider whether they should consider a bigger or smaller value, or even try them out and see the difference, then obviously they would be more likely to discover more about the relation between the different quantities in the problem.

They might even be prompted to consider a problem of the same structure but with the given value slightly changed, and spot the difference. According to the variation of learning by Marton et al. (2004), discernment, variation, and simultaneity are keys to learning new ways of seeing phenomena. As Marton pointed out, students cannot see the phenomenon (i.e., making sense of the problem) if they are concentrating on one part only of the picture. Students need to be able to look at the whole picture (part, part, and whole) before they can come to conceptual knowledge of the part-whole relationship. As Marton suggests, building-in variation can enable the student to look at the full picture. For example, when teaching missing addend problems, if the teacher changes the whole number (the cost of the book) and the known part (namely the number of coins already in the students' possession), and does this several times, the student has a much better chance of noticing the variation, discerning and paying attention to all the parts and, thereby focusing on them simultaneously. As Marton said, the learner should learn to discern certain necessary critical aspects in the phenomenon and focus on them simultaneously. When the child understands the phenomenon, the ability to select and fully apply the relevant procedural tools will help the child solve the problem in that particular context.

Being faced with a real-life problem makes the children address the issue of quantities as a real-life problem that they have to solve. They might not even think of it in terms of what they have learned at school – all they know is what they have – a quantity – 'not enough', what they have to spend – the price of the item – 'too much' or 'more than I have' and how much extra they need ('more'). They have the motivation to work out the problem. In the classroom, students tend to look at the problem with blinkers: they are asked to solve a particular problem, they hear the problem in words and they know they have to use a number sentence so they start to apply what the teacher has taught them – but it might be utterly meaningless to them unless they have had a similar experience outside school. In school, they are merely following a recipe. Outside school, they really have to think and be creative (Mohd, et.al., 2019).

5. Conclusion

This section discusses the various issues that have arisen from the study, as well as my conclusions, limitation, and suggestions for future research.

The issue got from the research indicates that some students just learn their mathematics as 'theory' in school. As I mentioned in the 'Difficulties in Making Sense of World Problems and Mathematical Knowledge' section, they may experience that world situation in ways different from us or in terms of the 'theory' we teach we think we have provided the sort of real-life situation which will allow them to apply our theory, such as: do I have enough money for this? Do I have enough money for that? How much change should I have? How much should I ask my mum to buy a certain thing, as I don't have enough? But they may have experienced such real-life situations differently, what that means in the word problems can be pure theory. They can't connect the theory in practice. The consequences of that word problem can't help them in real-life situations related to mathematics. For example, spending, and saving money, it also inhabits or restricts their knowledge in terms of problem-solving. Generally, it is not just related to mathematics; this could spill over into another area of learning. This could affect their ability to learn other disciplines such as history, chemistry, or subjects that are at the primary school level.

The usefulness of word problems in school can only be realized if the understanding of students on a certain situation can be elicited, to be enriched or embellished with their experience before that experience can be re-examined in the light of the theory that is applied to the real-life situation.

5.1. Research Limitations

Qualitative data is rich in depth and detail because it takes into account feelings, impressions, thoughts, and behavioral trends. This enables researcher to better understand their students' motivations and the factors that influence their practices. Because qualitative research is so in-depth, it takes a lot of time and resources, which limits the sample size. Even while extremely exact insights can be obtained, it is not practical to examine a large population.

Qualitative research has significant advantages in terms of adaptability and flexibility. Questions can be tailored to the information gathered, and the focus can easily be shifted to other topics of interest. However, when only a limited sample of thoughts from a qualitative study are available, generalizations about the greater community are challenging (Miles, et.al., 2014).

5.2. Ramifications for Teachers and Suggestions for Future Research

My findings have confirmed just how vital it is for teachers, when promoting conceptual understanding, to encourage students to look for and be mindful of relationships between different bits of knowledge and to give thought to what are the implications of any connection identified. They should also bear in mind the relationship between conceptual and procedural knowledge and use one to promote the other. In particular, it is important for teachers to help the students understand that an increase in conceptual knowledge can mean improvements in their procedural knowledge. Teachers should always be mindful to lead their students to make proper choices among the procedural tools available to them, and, where appropriate, to introduce the students to procedures.

Of particular importance, is the need for teachers (perhaps using the Socratic method of teaching) to stimulate their students' thought processes – to make them really think about the problem, question the way they have made sense of the problem, how they might look at it from a different viewpoint, how they might decide which procedure to use, whether or not their solution makes sense ('Does that look right?' You have written, '3+7 = 7' but think of it – if your mummy has given you three coins and your daddy gives you seven more, do you still only have seven left?').

Experienced teachers of mathematics understand that challenging a child's thought processes and procedures and encouraging a heuristic approach to problem-solving can reveal to students the techniques and pleasures of working it out for themselves - as Polya puts it: enjoying 'the triumph of discovery'. ('Teacher – I did it all by myself!!')

As experienced teachers know when a child works something out for themselves, they gain confidence in their own abilities, they become less afraid of problems, more interested and, as we all know, success breeds success. I hope that teachers can use my findings to help them determine the best way to help their students achieve their full potential. This research has presented the groundwork for finding the difficulties of 27 students. It would also be interesting to choose more students to do the research, or examining the same conceptual framework in a new context, location and/or culture.

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