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APPROXIMATE ADOMIAN SOLUTIONS TO THE BRIGHT OPTICAL SOLITARY WAVES OF THE CHEN-LEE-LIU EQUATION

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Abstract

The present paper examines the Chen–Lee–Liu (CLL) equation numerically. The equation is an important type of the Derivative Nonlinear Schrodinger (DNLS) equations that describe the propagation of pulse in optical fibers. The optical solitary wave solutions of the CLL equation comprises of the recently obtained bright solitons via ansatz method. However, in this paper, we employ the promising Adomian Decomposition Method (ADM) to further demonstrate its efficiency in solving different types of Schrodinger equations. The method reveals amazing

approximate solutions with minimal error as shown in the respective comparison tables and various plots depicted.

Keywords

Optical Solitary Waves, Adomian Decomposition Method, Numerical Method, Chen-Lee-Liu Equation

1. Introduction

Nonlinear Schrödinger equations are important class of evolution equations with wide range of applications including pulse propagation in networking and in many communication industries. A typical example of nonlinear Schrödinger equation with great applications in optical pulse propagation in monomode and optical fibers among others is the CLL equation that emerged in 1979 (Chen, Lee & Liu, 1979). In recent years, there has been an increasing interest with regards to optical solitons due to their higher need in networking sectors (Agrawal, 1997). These solitons are normally revealed in hyperbolic functions with different forms including the singular, dark, bright, combo and w-shaped soliton forms among others (Triki, Zhou, Moshokoa, Ullah, Biswas & Belic, 2018), (Aliyu, Inc, Yusuf, Bayram, Baleanu, 2019), (Triki, Hamaizi, Zhou, Biswas, Ullah, Moshokoa & Belic, 2018), (Triki, Hamaizi, Zhou, Biswas, Ullah, Moshokoa & Belic, 2018). However, there is little published works on optical solitons of CLL equation using numerical techniques, see (González-Gaxiola, & Biswas, 2018) and (Mohammed, Bakodah, Banaja, Alshaery, Zhou, Biswas & Belic, 2019). Thus, the present work as a goal will utilize the efficient ADM (Adomian, 1994) being an established powerful and effective numerical tool for decades to numerically investigate the CLL equation. Furthermore, to achieve the set objectives, the newly presented bright optical solitons (Triki, Babatin & Biswas, 2017) would be sought as benchmark solutions for comparative studies. Also the error plots would be presented in this paper for each of these soliton cases. Also, an error analysis will be carried out in order to assess the effectiveness of the used method. Accordingly, we consider the CLL equation (Chen, Lee, & Liu, 1979), (Agrawal, 1997), (Triki, Zhou, Moshokoa, Ullah, Biswas & Belic, 2018), (Aliyu, Inc, M., Yusuf, Bayram, & Baleanu, 2019), (Triki, Hamaizi, Zhou, Biswas, Ullah, Moshokoa & Belic, 2018), (Triki, Hamaizi, Zhou, Biswas, Ullah, Moshokoa & Belic, 2018), (González-Gaxiola & Biswas, 2018), (Mohammed, Bakodah, Banaja, Alshaery, Zhou, Biswas,... & Belic, 2019), (Triki, Babatin & Biswas, 2017) of the form

$$iu_t + au_{xx} + ib|u|^2u_x = 0, \qquad (1)$$

where the complex-valued function u depends on the space and time variables x and t; and the self-steeping phenomena and group dispersion parameters (nonzero) a and b, respectively. It is also pertinent to note that Eq. (1.1) reduces to a Regular Chen-Lee-Liu (RCLL) equation while setting $a = \mathrm{b} = 1$ (Rogers & Chow, 2012). See also (Kara, Biswas, Zhou, Moraru, Moshokoa & Belic, 2018), (Al Qarni, Banaja, Bakodah, Alshaery, Majid, & Biswas, 2016), (Sadeeg, Nuruddeen & Gomez-Aguilar 2019), (Nuruddeen, 2017), (Wazwaz, 2000), (Jawad, Biswas, Zhou, Alfiras, Moshokoa & Belic, 2018), (Kivshar & Agrawal, 2003), (Bakodah, Al Qarni, Banaja, Zhou, Moshokoa, & Biswas, 2017), (Wazwaz, M. 2010) and the references therein for the related literature on the general Schrödinger equations' methods and solitary waves.

2. The Bright Optical Solitary Waves

In this section we present the newly constructed bright solitary wave solutions for the CLL equation given in Eq. (1) by (Triki, Babatin & Biswas, 2017) using the ansatz method. These bright solitons to be considered would later be used as benchmark solutions for numerical comparisons.

i. The first bright soliton solution to be considered for $\delta < 0$ and $\sigma > 0$ is given by (Triki, Babatin & Biswas, 2017) is:

$$u(x,t) = A\frac{\operatorname{sech}(\eta s)}{\sqrt{1+B\,sech^2(\eta s)}}e^{i[-kx+\omega t+\theta(s)]}, \qquad (2)$$

where η and A are the inverse width and amplitude for the soliton, with the following

$$A^2 = -\frac{2\delta(2B+1)}{\sigma}, \quad \eta^2 = -\delta, \quad 2B+1 = \pm\left(1-\frac{16\gamma\delta}{3\sigma^2}\right)^{-\frac{1}{2}} \text{ and } \gamma < \left|\frac{3\sigma^2}{16\delta}.\right| \qquad (3)$$

ii. The second bright soliton solution for $\delta > 0$ and $\sigma < 0$ is given by (Triki, Babatin & Biswas, 2017) is:

$$u(x,t) = P\sqrt{1+\operatorname{sech}(\mu s)}\; e^{i[-kx+\omega t+\theta(s)],} \qquad (4)$$

where μ and P are given by the following

$$\mu^2 = \frac{4\delta}{5},\ P^2 = -\frac{8\delta}{5\sigma} \quad \text{and } \gamma = \frac{15\sigma^2}{64\delta}. \qquad (5)$$

iii. The third bright soliton solution for $\delta < 0$ and $\sigma > 0$ is given by (Triki, Babatin & Biswas, 2017) :

$$u(x,t)=\frac{P}{\sqrt{1+r\cosh(\rho s)+\lambda\sinh(\rho s)}}e^{i[-kx+\omega t+\theta(s)]}, \quad (6)$$

where P, ρ and r are given by the following

$$P^2=-\frac{4\delta}{\sigma},\ \ \rho^2=-4\delta,\ \ r^2=1+\lambda^2-\frac{16\gamma\delta}{3\sigma^2},\ \ \text{and}\ \ \gamma<\left|\frac{3\sigma^2(1+\lambda^2)}{16\delta}\right|. \quad (7)$$

3. Approximate Adomian Solutions

In this section, we will apply the principle of the ADM (Adomian, 1994) for the determination of the overall recursive scheme for the CLL equation given in Eq. (1).
First, we rewrite the CLL equation using operator notation as follows:

$$L_t u=aiu_{xx}-b|u|^2u_x, \quad (8)$$

where $L_t=\frac{\partial}{\partial t}$ is the linear operator in t and its corresponding inverse operator defined by $L_t^{-1}=\int_0^t(.)dt$ t. Now applying this inverse operator to Eq. (8) we obtain

$$u=aiL^{-1}u_{xx}-bL^{-1}A, \quad (9)$$

where the nonlinear term in Eq. (9) is represented by the following

$$A=|u|^2u_x. \quad (10)$$

The decomposition method (Adomian, 1994) gives solution by infinite summation of series of the following form

$$u(x,t)=\sum_{n=0}^{\infty}u_n(x,t), \quad (11)$$

and the nonlinear term in Eq. (10) is decomposed as

$$A=\sum_{n=0}^{\infty}A_n, \quad (12)$$

where A_n are Adomian's polynomials, which are given by

$$A_n=\frac{1}{n!}\frac{d^n}{d\lambda^n}N\left(\sum_{i=0}^{\infty}(\lambda^i u(x,t))\right)_{\lambda=0},\quad n=0,1,2,\ldots \quad (13)$$

Substituting Eqs. (11) and (12) into Eq. (9), we get

$$\sum_{n=0}^{\infty}u_n(x,t)=aiL^{-1}\sum_{n=0}^{\infty}u_{n_{xx}}(x,t)-bL^{-1}\sum_{n=0}^{\infty}A_n. \quad (14)$$

Consequently, the overall recurrent formula for the CLL the ADM idea is determined as follows:

$$\begin{cases} u_0(x,t) = u(x,0), \\ u_{k+1}(x,t) = aiL^{-1}\left(u_{k_{xx}}(x,t)\right) - bL^{-1}A_k, \quad k \geq 0 \end{cases} \quad (15)$$

4. Numerical Results and Analysis

The present section gives some computational results using the overall recursive scheme given in Eq. (15) for the CLL equation. The soliton solutions given earlier in the second section will be considered as benchmark exact analytical solutions for comparative analysis. The error analysis in each type is presented through tables 1-3 and shown in figures 1-9 respectively as follows

Table 1: *The Absolute Error of Numerical Method for the First Soliton with* $a = 0.03$

x	$b = -10$		$b = -1$		$b = -0.1$	
	$t = 0.2$	$t = 0.5$	$t = 0.2$	$t = 0.5$	$t = 0.2$	$t = 0.5$
	$\lvert u_{Exact} - u_{ADM}\rvert$					
-2	$9.935\ 10^{-9}$	$2.483\ 10^{-8}$	$3.141\ 10^{-8}$	$6.283\ 10^{-8}$	$9.935\ 10^{-8}$	$2.483\ 10^{-7}$
-1	$9.936\ 10^{-9}$	$2.484\ 10^{-8}$	$3.142\ 10^{-8}$	$6.284\ 10^{-8}$	$9.936\ 10^{-8}$	$2.484\ 10^{-7}$
1	$9.936\ 10^{-9}$	$2.484\ 10^{-8}$	$3.142\ 10^{-8}$	$6.284\ 10^{-8}$	$9.936\ 10^{-8}$	$2.484\ 10^{-7}$
2	$9.935\ 10^{-9}$	$2.483\ 10^{-8}$	$3.141\ 10^{-8}$	$6.283\ 10^{-8}$	$9.935\ 10^{-8}$	$2.483\ 10^{-7}$

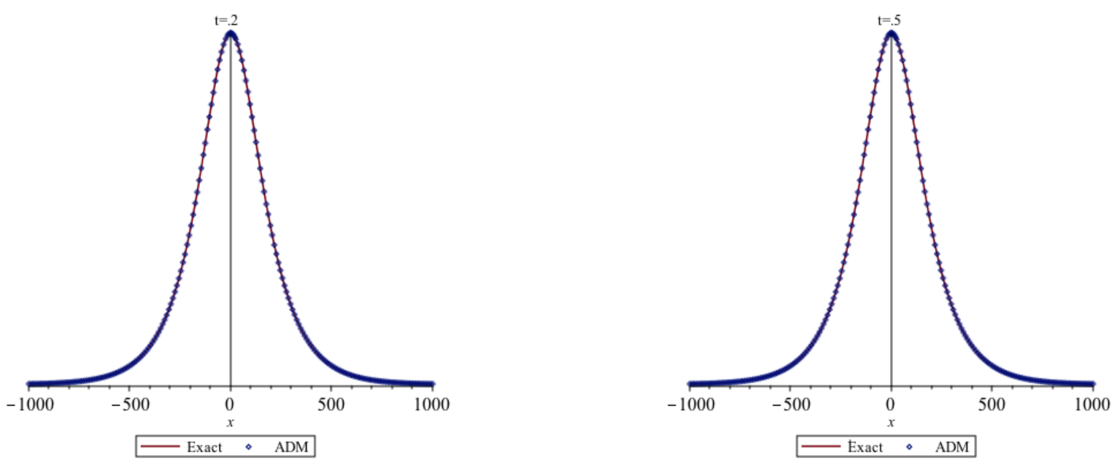

Figure 1: *Comparison between Approximate Solution and Exact Solution for the First Soliton Solution with* $a = 0.03$ and $b = -10$

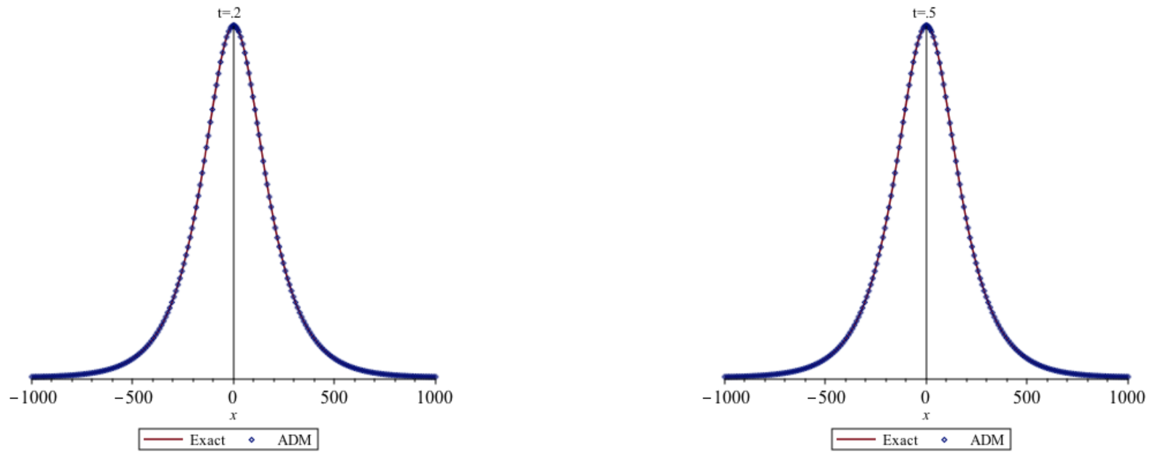

Figure 2: *Comparison between Approximate Solution and Exact Solution for the First Soliton Solution with* $a = 0.03$ and $b = -1$

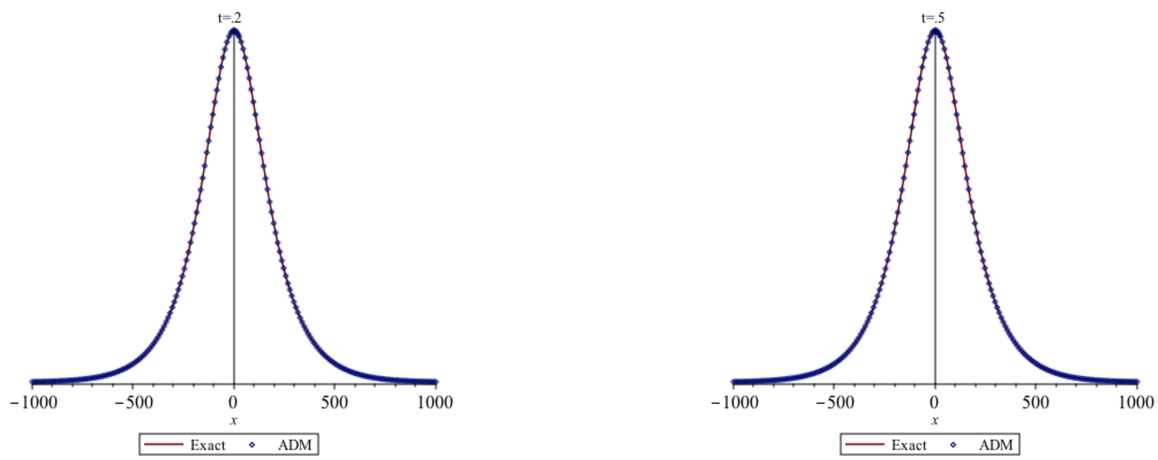

Figure 3: *Comparison between Approximate Solution and Exact Solution for the Soliton Solution with* $a = 0.03$ and $b = -0.1$

Table 2: *The Absolute Error of Numerical Method for the Second Soliton with* $a = 0.5$

	$\boldsymbol{b = 10}$		$\boldsymbol{b = 1}$		$\boldsymbol{b = 0.1}$	
$\boldsymbol{x}$	$\boldsymbol{t = 0.2}$	$\boldsymbol{t = 0.5}$	$\boldsymbol{t = 0.2}$	$\boldsymbol{t = 0.5}$	$\boldsymbol{t = 0.2}$	$\boldsymbol{t = 0.5}$
	$\lvert u_{Exact} - u_{ADM} \rvert$					
$\boldsymbol{-2}$	$3.52\ 10^{-5}$	$8.82\ 10^{-5}$	$1.12\ 10^{-4}$	$2.79\ 10^{-4}$	$3.53\ 10^{-4}$	$8.82\ 10^{-4}$
$\boldsymbol{-1}$	$3.52\ 10^{-5}$	$8.82\ 10^{-5}$	$1.12\ 10^{-4}$	$2.79\ 10^{-4}$	$3.53\ 10^{-4}$	$8.82\ 10^{-4}$
$\boldsymbol{1}$	$3.52\ 10^{-5}$	$8.82\ 10^{-5}$	$1.12\ 10^{-4}$	$2.79\ 10^{-4}$	$3.53\ 10^{-4}$	$8.82\ 10^{-4}$
$\boldsymbol{2}$	$3.52\ 10^{-5}$	$8.82\ 10^{-5}$	$1.12\ 10^{-4}$	$2.79\ 10^{-4}$	$3.53\ 10^{-4}$	$8.82\ 10^{-4}$

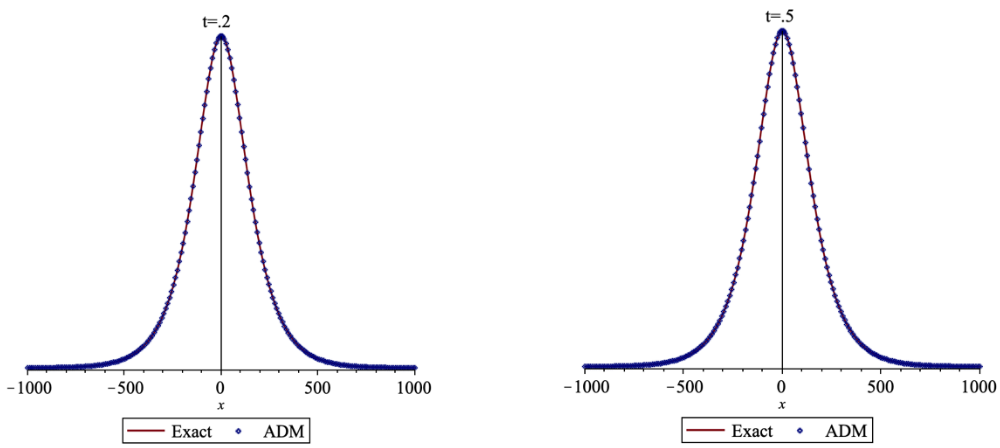

Figure 4: *Comparison between Approximate Solution and Exact Solution for the Second Soliton Solution with* $a = 0.5$ and $b = 10$

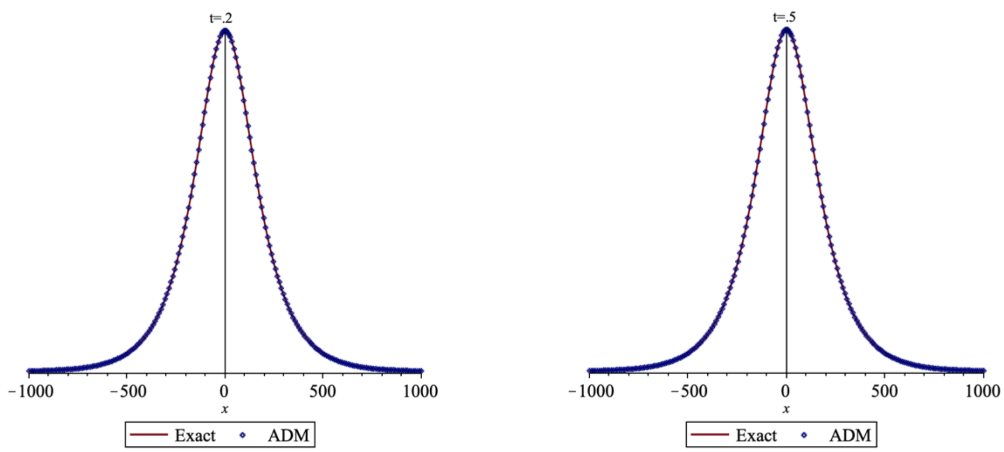

Figure 5: *Comparison between Approximate Solution and Exact Solution for the Second Soliton Solution with* $a = 0.5$ and $b = 1$

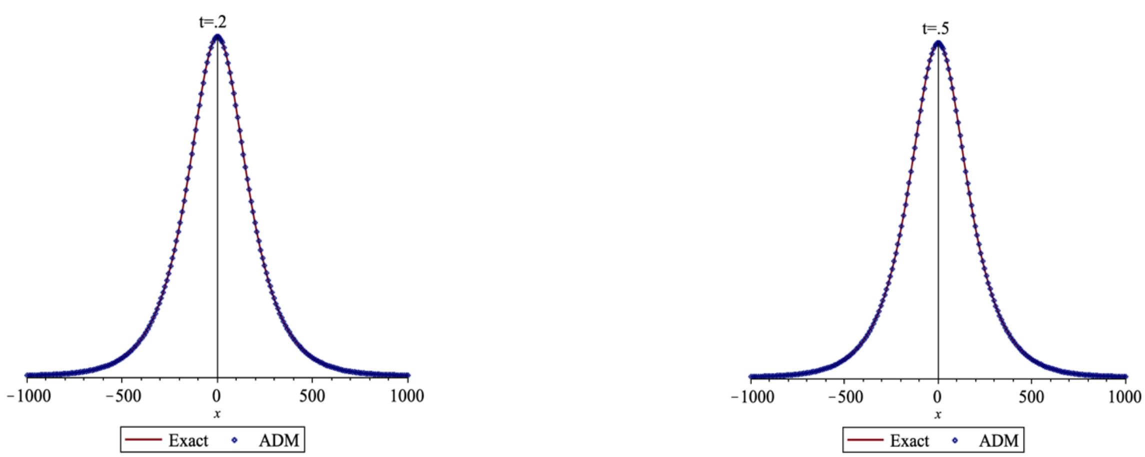

Figure 6: *Comparison between Approximate Solution and Exact Solution for the Second Soliton Solution with* $a = 0.5$ and $b = 0.1$

Table 3: *The Absolute Error of Numerical Method for the Third Soliton with* $a = 0.01$

x	$b = -10$		$b = -1$		$b = -0.1$	
	$t = 0.2$	$t = 0.5$	$t = 0.2$	$t = 0.5$	$t = 0.2$	$t = 0.5$
	$\lvert u_{Exact} - u_{ADM} \rvert$					
-2	$7.73\ 10^{-8}$	$1.93\ 10^{-7}$	$7.76\ 10^{-7}$	$1.94\ 10^{-6}$	$8.65\ 10^{-6}$	$2.16\ 10^{-5}$
-1	$8.07\ 10^{-8}$	$2.02\ 10^{-7}$	$8.07\ 10^{-7}$	$2.02\ 10^{-6}$	$8.31\ 10^{-6}$	$2.08\ 10^{-5}$
1	$8.05\ 10^{-8}$	$2.01\ 10^{-7}$	$8.06\ 10^{-7}$	$2.02\ 10^{-6}$	$8.33\ 10^{-6}$	$2.08\ 10^{-5}$
2	$7.70\ 10^{-8}$	$1.93\ 10^{-7}$	$7.74\ 10^{-7}$	$1.94\ 10^{-6}$	$8.68\ 10^{-6}$	$2.17\ 10^{-5}$

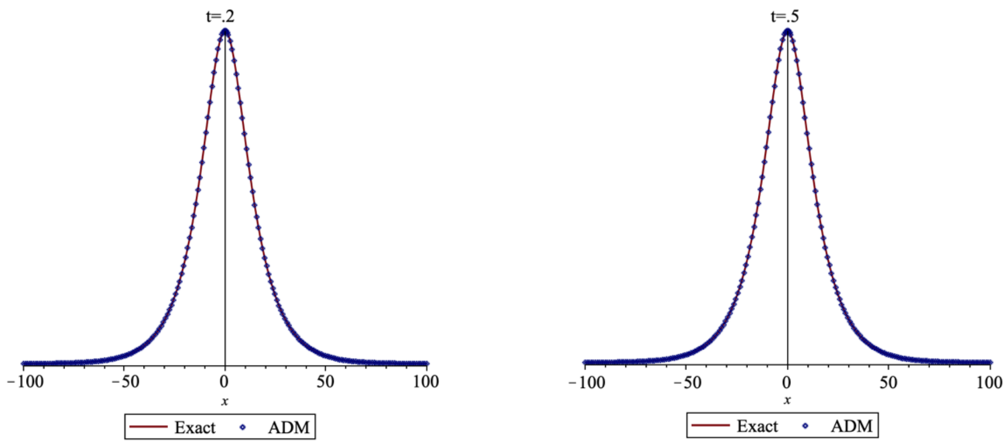

Figure 7: *Comparison between Approximate Solution and Exact Solution for the Third Soliton Solution with* $a = 0.01$ and $b = -10$

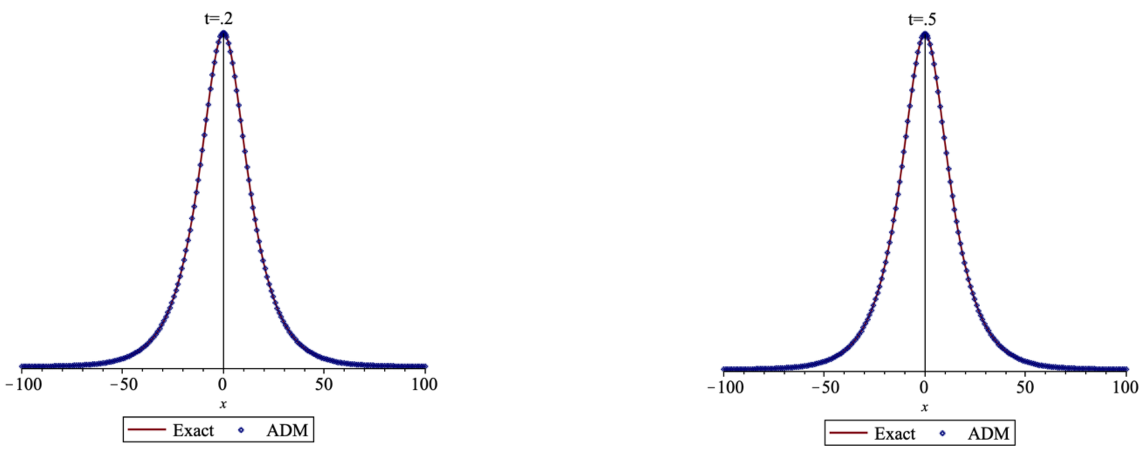

Figure 8: *Comparison between Approximate Solution and Exact Solution for the Third Soliton Solution with* $a = 0.01$ and $b = -1$

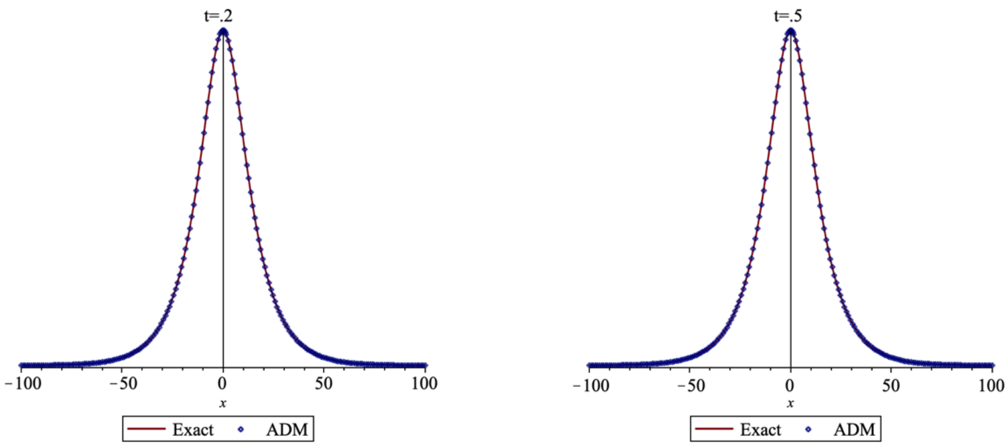

Figure 9: *Comparison between Approximate Solution and Exact Solution for the Third Soliton Solution with* $a = 0.01$ and $b = -0.1$

5. Conclusion

In conclusion, this paper recovered the three cases of bright solitary wave solutions for the CLL equation that were recently presented numerically with the aid of the ADM. The obtained numerical results demonstrate excellent agreement with the benchmark bright soliton solutions considered. Error analysis and several plots are also presented. The results of this paper will be useful in optical fibers and communication among others. Furthermore, future studies also will be addressed the dark and w-shaped optical solitons of CLL equation using some integration schemes.

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