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ON GENERALIZATION OF EXTENDING ACTS AND M-JECTIVE ACTS

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Abstract

An S-act M_s is called a generalized extending act (for short a GE-act) if the following condition is satisfied: If $M_s = M_1 \dot{\cup} M_2$, and X is subact of M_s, then there exist C_i is a retract of M_i (i = 1, 2) such that $C_1 \dot{\cup} C_2$ is a complement of X in M_s. In this article, the notion of generalized extending S-act is introduced and studied as a concept of generalizing extending act which was presented by the author. Some properties of such acts in analogy with the known properties for extending acts are illustrated .Besides, the author has introduced in a diagram of acts and homomorphisms, the concept of generalized of quasi injective which is also representing a generalization of M-injective acts. Here we introduce the concept of M-jective acts, which is a generalization of the concept of M-injectivity. An S-act Y is called X-jective if every complement Z of Y in M_s is a retract, where $M_s = X \dot{\cup} Y$. The concept of M-jective acts is used here to solve the problem of finding a necessary and sufficient condition for a direct sum of extending acts to be extending. Indeed, we show that relative jectivity is necessary and sufficient for a direct sum of two extending acts to be extending as in module theory. Some properties and characterizations of generalizing extending act and M-jective act are illustrated. Conditions on which subact inherit the property of generalizing extending act were demonstrated. The

relationship among extending act and generalizing extending act, act with condition (C_1^*) and generalizing extending act was elucidated. Conclusions and discussion of this work were clarified in the last section.

Keywords

Generalization of Extending Acts, M-JECTIVE ACTS, Direct Sums of Uniform Acts, Indecomposable Acts, Absolute Relative Jective Act (ARJ-act), Extending Acts

1. Introduction

In (Shaymaa A., 2018), (Abbas M.S. and Shaymaa A., 2015), (Shaymaa A., 2015), (Shaymaa A., 2016), (Abbas M.S. and Shaymaa A., 2015), (Abbas M.S. and Shaymaa A., 2016), (Shaymaa A., 2016), (Abbas M.S. and Shaymaa A., 2015), the author has introduced about the generalizations in systems over monoids, and the concept of QP-injective act, Generalizations of quasi injective acts over monoids, Pseudo C-M-injective and pseudo C-quasi principally injective acts over monoids, Pseudo injective and pseudo-QP-injective S-systems over monoids, Pseudo Finitely Quasi-injective systems over monoids, Finitely Quasi injective and Quasi finitely injective S-systems over monoids, Pseudo PQ-injective systems over monoids, etc. which is generalized of quasi injective and thereby it is generalized of M-injective acts. The principal objectives of the study were as follows.

- To introduce and study the generalizing of extending act.
- To introduce and study M-jectivity for solving the problem of direct sum of extending acts.

For this reason, the research is to introduce the concept of M-jectivity, which is a generalization of M-injective. In the present article, the author presents the new concept and we require that every complement of A in $M\dot{\cup}A$ is a retract and need not have a specific complementary retract in $M\dot{\cup}A$. Indeed, S-act A_s is M -jective if every complement of A_s in $M\dot{\cup}A$ is a retract. If A_s is M-jective and M_s is A-jective, we say that A_s and M_s are relatively jective. The problem of finding a necessary and sufficient condition for a direct sum of extending acts to be extending is still open problem. It has been investigated in an article by (Shaymaa A., 2017), that relative injectivity is sufficient but not necessary such that the author was shown that a direct sum of extending acts M_1 and M_2 is extending if and only if every closed subact with zero intersection with M_1 or with M_2 is a retract (proposition 2.11) in (Shaymaa A., 2017).

In this work, we show that relative jectivity is necessary and sufficient for a direct sum of two extending acts to be extending. We also introduce the concept of generalized extending acts, and give some properties of such acts which are analogous to the properties which are known for extending acts.

By an act M_s we mean a unitary right act over monoids. Note that we utilized the terminology and notations from (Shaymaa A., 2017) and (Shaymaa A., 2016) freely.

A proper subact N of an S-act M_s is called maximal if for each subact K of M_s with $N \subseteq K \subseteq M_s$ implies either K = N or $K = M_s$. (Shaymaa A., 2015).

If X and Y are subacts of M_s respectively, then X is a complement of Y in M_s if X is a maximal in M_s with the property that $X \cap Y = \Theta$. It is obvious that every complement in M_s is a closed subact of M_s .

Let A_s , M_s be two S-systems . A_s is called M-injective if given an S-monomorphism $\alpha: N \longrightarrow M_s$ where N is a subsystem of M_s and every S-homomorphism $\beta: N \rightarrow A_s$, can be extended to an S-homomorphism $\sigma: M_s \longrightarrow A_s$ (Berthiaume P.,1967). An S-system A_s is injective if and only if it is M-injective for all S-systems M_s.

An S-act M_s is extending (or a CS-act, or act with (C_1)) if every subact is essential in a retract (or equivalently, if X is subact of M_s , then there is a decomposition $M_s = M_1 \dot{\cup} M_2$ such that X is subact of M_1 and $X \dot{\cup} M_2$ is essential in M)(Shaymaa A.,2017). Extending acts generalize quasi-injective acts.

The between brackets equivalent defining condition for extending acts can be generalized to the following condition:

(C_1^*) If X is subact of M_s, then there is a decomposition $M_s = M_1 \dot{\cup} M_2$ such that $X \cap M_2 = \Theta$ and $X \dot{\cup} M_2$ is essential subact of M_s.

It is obvious that every extending act satisfy condition(C_1^*). The present work consists of two sections. The first one (section two) is devoted to introduce and investigate a new kind of generalization M-injective S-acts, namely M-jective acts. Properties and characterizations of these S-acts are investigated. The other section (section three), is focused on generalized extending acts. We have demonstrated that every extending act is a GE- act, and also gave the relationship between acts with (C_1^*) and GE-acts. Conditions under which subacts are inherited the property of generalized extending acts are elucidated.

It is well-known that communicating research findings in science education and technology to witness lots of complexities with respect to the terms, methods, and language of communicating the results (John Olakunle Babayemi, 2017). Besides, it is essential and important to study the underlying relationship between internet self-efficacy and interaction in Mathematics courses (Remelyn L. Asahid, 2018).In fact, interaction creates a major environment in learning Mathematics effectively. If we take the interaction from another side between the writer of the article and the reviewer and how one can send him the information palpably? For this reason, we tried to display this work without ambiguous.

2. M-Jective Acts

As a generalization of M-injective acts, we introduce the following definition:

Definition (2.1): Let $M_s = X\dot{\cup}Y$. Then Y is called X-jective if every complement Z of Y in M_s is a retract.

Lemma (2.2): Let X and Y be subacts of act M_s with $X\cap Y = \Theta$. Then X is a complement of Y in M_s if and only if X is a closed subact of M_s and $X \dot{\cup} Y$ is essential in M_s.

Proof: $\Longrightarrow$)Let X and Y be subacts of S-act M_s with $X\cap Y = \Theta$ and X is a complement of Y in M_s, then by prop.(2.6) in(A.Shaymaa,2017) , $X\dot{\cup}Y$ is $\cap$-large in M_s this implies that X is a retract of M_s.Thus by remarks and examples (2.2) in(A.Shaymaa, 2016) X is closed.

$\Longleftarrow$)It is obvious.■

Lemma (2.3) Let $M_s = A\dot{\cup}B$. Let C be a complement in A of a subact X of A. Then:

(1)$C\dot{\cup}B$ is a complement of X in M_s.

(2)C is a complement for $X\dot{\cup}B$ in M_s.

Proof: (1) Let $C\dot{\cup}B$ be subact of S-act Y and Y be subact of M_s such that $Y\cap X = \Theta$. Since $(Y\cap A)\cap X = \Theta$, and C is a complement of X in A , it is follows that $Y\cap A = C$; and hence $Y=B\dot{\cup}(Y\cap A)=B\dot{\cup}C$.

(2) The fact the C is a complement of Y in A implies that $C\dot{\cup}Y\dot{\cup}B$ is ⋂-large in M_s . It is clear that if C is closed in A and A is a retract of M_s (A is closed in M_s), then C is closed in M_s . Thus, by lemma 2.2, and since C is closed in A (hence in M_s), C is a complement of $X\dot{\cup}B$ in M_s.■

Proposition (2.4): Let $M_s = X \dot{\cup} Y$, where Y is X-jective. Let $X = X_1 \dot{\cup} X_2$, and $Y = Y_1 \dot{\cup} Y_2$. Then (for i, j = 1, 2):

(1)Y_i is X-jective;

(2)Y is X_j-jective;

(3)$Y_i$is X_j-jective.

Proof: (1) Write $M_s=X \dot{\cup} Y_1 \dot{\cup} Y_2$. Let C be a complement of Y_1 in $X\dot{\cup}Y_1$. Then by (2) of lemma (2.3), C is a complement of Y in M_s . Since Y is X-jective, then C is a a retract.

(2) Write $M_s= X_1 \dot{\cup} X_2 \dot{\cup} Y$. Let C be a complement of Y in $X_1\dot{\cup}Y$. Then by (1) of lemma (2.3), $C\dot{\cup}X_2$ is a complement of Y in M_s . Since Y is X-jective, $C\dot{\cup}X_2$ is a retract, and hence C is a retract of $X_1\dot{\cup}Y$

(3) Follows from (1) and (2). ■

Lemma (2.5): Let $M_s= X \dot{\cup} Y$, where Y is X-jective. If X is extending, then every closed subact C of M_s, with C⋂Y = Θ, is a retract of M_s.

Proof: Since X is an extending act, we have $(C \dot{\cup} Y) \cap X$ is ⋂-large subact of X_1 where X_1 is a retract of X, and hence $((C \dot{\cup} Y) \cap X) \dot{\cup} Y$ is ⋂-large subact of $X_1\dot{\cup}Y$. Since $C\dot{\cup}Y=((C \dot{\cup} Y)\cap X)\dot{\cup}Y)$, we have $(C\dot{\cup}Y)$ is ⋂-large subact of $X_1\dot{\cup}Y$. By lemma (2.2), C is a complement of Y in $X_1\dot{\cup}Y$. By proposition (2.4) implies that Y is X_1-jective. Therefore C is a retract of $X_1\dot{\cup}Y$ where $X_1\dot{\cup}Y$ is a retract of M_s.■

Lemma (2.6): (proposition 2.11 in (A. Shaymaa, 2017)) Let $M_s= M_1\dot{\cup}M_2$, where M_1 and M_2 are both extending acts. Then, M_s is extending if and only if every closed subact N of M_s with $N\cap M_1= \Theta$ or $N\cap M_2= \Theta$ is a retract of M_s.■

The following is a necessary and sufficient condition of a direct sum of two extending acts to be extending.

Theorem (2.7): Let $M_s= M_1\dot{\cup}M_2$. Then M_s is extending if and only if the M_i is extending, and is M_j-jective, if $i\neq j(= 1, 2)$.

Proof: Follows from lemma 2.5, and lemma 2.6.■

Corollary (2.1): An S-act M_s with the condition (C_1^*) is extending if and only if M_s has the property that X is Y-jective for every decomposition of $M_s = X \dot{\cup} Y$.

Proof: By the condition (C_1^*), every closed subact of M_s is a complement of a retract of M_s . Hence, by assumption, every closed subact is a retract. Therefore M_s is extending. The converse is obvious.■

3. Generalized Extending Acts

Definition (3.1): An S-act M_s is called a generalized extending act (for short a GE-act) if the following condition is satisfied: If $M_s = M_1 \dot{\cup} M_2$, and X is subact of M_s , then there exist C_i is a retract of M_i (i = 1, 2) such that $C_1 \dot{\cup} C_2$ is a complement of X in M_s.

It is clear that every extending act is GE-act, but the converse is not true in general for example the Z-act $M_s = Z_2 \dot{\cup} Z$ is a GE-act, while M_s is not an extending act (by corollary (3.6) below).

Note that in condition (C_1^*) and according to lemma (2.2), we have M_2 is a complement of X in M_s . Thereby, condition (C_1^*) is equivalent to the following: every subact has a complement in M_s which is a retract. Also, from definition (3.1) every GE-act is satisfying condition (C_1^*).

In the following, we will demonstrate that every extending act is a GE- act, and also give the relation between acts with (C_1^*) and GE-acts.

Lemma (3.1): The following are equivalent for an S-act $M_s = X \dot{\cup} Y$:

(1) X has (C_1^*);

(2) For every closed subact N of M_s , with $N \cap Y = \Theta$, there exists X_1 is a retract of X such that $X_1 \dot{\cup} Y$ is a complement of N in M_s .

Proof: (1) ⟹ (2) Let N be a closed subact of M_s , with $N \cap Y = \Theta$. By the condition (C_1^*) for X, there exists X_1 is a retract of X such that X_1 is a complement of $(N \dot{\cup} Y) \cap X$ in X. As $[(N \dot{\cup} Y) \cap X] \dot{\cup} X_1$ is ∩-large in X, we have that $[(N \dot{\cup} Y) \cap X] \dot{\cup} X_1 \dot{\cup} Y$ is ∩-large in M_s. Since

$N\dot{\cup} Y = [(N\dot{\cup} Y) \cap X]\dot{\cup}Y$, it follows that $N\dot{\cup}Y\dot{\cup}X_1$ is $\cap$-large in M_s. Thus, by lemma (2.2), $X_1\dot{\cup} Y$ is a complement of N in M_s.

(2)⟸(1) Let N be a closed subact of X. Since a closed subact in a retract of M_s is closed in M_s, it follows that N is closed in M_s. By (2), there exists X_1 is retract of X such that $X_1\dot{\cup} Y$ is a complement of N in M_s. It follows that $N \dot{\cup} X_1\dot{\cup} Y$ is $\cap$-large in $M_s= X\dot{\cup}Y$ and hence $N \dot{\cup}X_1$ is $\cap$-large in X. By lemma (2.2), X_1 is a complement of N in X, and therefore X has (C_1^*).■

The author explained previously that direct sums of two extending acts need not be extending (A.Shaymaa, 2017). In the following theorem we will illustrate that direct sums of two acts with (C_1^*) are acts with (C_1^*).

Theorem (3.2): If $M_s = M_1\dot{\cup}M_2$, where M_1 and M_2 are both have the condition (C_1^*), then M_s has (C_1^*).

Proof: Let N be a closed subact of M_s, and let N_1 be a maximal essential extension of $N\cap M_1$ in N. It is clear that N_1 is closed in M_s with $N_1\cap M_2= \Theta$. Hence by lemma (3.1), there exists a complement of N_1 in M_s of the form $N_1\dot{\cup}M_2$ such that N_1 is a retract of M_1. As $N_1\dot{\cup}H_1\dot{\cup}M_2$ is $\cap$-large in M_s, we have that $N_1\dot{\cup}[N\cap(H_1\dot{\cup}M_2)]$ is $\cap$-large in N. Let N_2 be a maximal essential extension of $N\cap(H_1\dot{\cup}M_2)$ in N. It is clear that N_2 is a closed subact of M_s with $N_2\cap M_1= \Theta$ (due to $N\cap(H_1\dot{\cup}M_2) \cap M_1 = N\cap H_1$ is subact of N_1. Hence, again by lemma (3.1), there exists a complement of N_2 in $M_s$of the form $M_1\dot{\cup}H_2$ such that H_2 is a retract of M_2. It is easy to see that the sum $N_1\dot{\cup}N_2\dot{\cup}H_1\dot{\cup}H_2$ is a direct sum. Since $(N \cap M_1) \dot{\cup}H_1\dot{\cup}N_2\dot{\cup}H_2$ is $\cap$-large in $M_1\dot{\cup}N_2\dot{\cup}H_2$ is $\cap$-large in M_s, it follows that $N\dot{\cup}H_1\dot{\cup}H_2$ is $\cap$-large in M_s, and thus, by lemma(2.2), $H_1\dot{\cup}H_2$is a complement of N in M_s. Therefore N has a complement in M_s which is a retract of M_s.■

Corollary (3.3): The following are equivalent for an S-act M_s:

(1)M_s is a GE-act.

(2)Every retract of M_s has (C_1^*).■

Corollary (3.4): Retract of GE-acts are GE-acts.

Proof: Is an immediate consequence of corollary (3.3).■

Corollary (3.5): Every extending act is a GE-act.

Proof: Since every retract of an extending act is extending, hence has (C_1^*).■

Corollary (3.6): The following are equivalent for an S-act $M_s = \dot{\bigcup}_{i=1}^{n} M_i$

(1)The M_i has the condition (C_1^*) , where i=1,2,…,n.

(2)Each closed subact of M_s has a complement in M_s of the form $\dot{\bigcup}_{i=1}^{n} N_i$, where N_i is a retract of M_i(i=1,2,…,n).

Proof: (1)⟹(2) By induction on the number n of the retracts M_i of M_s , and by using that a complement of arbitrary closed subact has the form $N_1 \dot{\cup} N_2$, where N_i is a retract of M_i(i=1,2) and $M_s = M_1 \dot{\cup} M_2$.

(2)⟸(1) Follows from the fact that each closed subact of M_i is closed in M_s.■

Definition (3.7): An S-act M_s is called an absolute relative jective act (for short ARJ-act) if M_i is M_j-jective ($i \neq j$); whenever $M_s = M_1 \dot{\cup} M_2$.

It is clarified that every extending act is an ARJ-act (theorem 2.7), but the converse is not true in general for example: any indecomposable act is an ARJ-act, which is not extending. The following proposition gives a condition under which extending acts and ARJ-acts are equivalent

Proposition (3.8): The following are equivalent for an S-act M_s:

(1)M_s is an extending act;

(2)M_s is an ARJ-act and satisfies the condition (C_1^*).

Proof: (1)⟹(2) From theorem(2.7), and since extending acts satisfy the condition (C_1^*).

(2)⟸(1) Let N be a closed subact of M_s . By the condition (C_1^*), we have that N has a complement in M_s which is a retract; i.e. M_s has a decomposition M_s=$M_1 \dot{\cup} M_2$, where $M_2 \dot{\cup} N$ is $\cap$-large subact in M_s. Since M_s is an ARJ-act, M_2 is M_1-jective. From lemma (2.2) N is a complement of M_2 in M_s , and hence from the definition of relative jectivity, N is a retract subact of M_s. Therefore M_s is extending.■

Proposition (3.9): Every indecomposable act M_s with the condition (C_1^*) is uniform.

Proof: Let N be a nonzero subact of M_s. By (C_1^*), there exists a decomposition of M_s as $M_s = M_1 \dot{\cup} M_2$ such that $N \dot{\cup} M_2$ is ⋂-large subact inM_s. Since M_s is indecomposable, we have $M_2 = \Theta$; and hence N is ⋂-large subact in M_s.■

Definition (3.10): In the context of act theory, the socle of S-act M_s is the set:

Soc(M)= ⋂ {A | A is ⋂-large subact of M_s}.

Proposition (3.11): If M_s has (C_1^*), then it has a decomposition $M_s = M_1 \dot{\cup} M_2$, where Soc(M_s) is ⋂-large subact in M_1.

Proof: By (C_1^*), there exists a subact M_2 of M_s such that $M_s = M_1 \dot{\cup} M_2$, and Soc(M_s)$\dot{\cup} M_2$ is ⋂-large subact in M_s . It is obvious that Soc(M_2) = Θ, and Soc(M_s) is ⋂-large subact in M_1.■

The following proposition explains that arbitrary direct sums of uniform acts must have (C_1^*).

Proposition (3.12): Direct sums of uniform acts have (C_1^*).

Proof: Let $M_s = \dot{\cup}_{i=1}^{n} N_i$, where the Ni are uniforms, and let X be a subact of M_s. By Zorn's lemma, there exists J ⊆ I maximal with respect to X⋂($\dot{\cup} N_i$) = Θ . Since X$\dot{\cup}$($\dot{\cup}_{i \in j} N_i$) is ⋂-large subact in M_s. It implies that by lemma (2.2) $\dot{\cup}_{i \in j} N_i$ is a complement of X in M_s .■

Lemma (3.13): Let X be subact of Y and Y be subact of S-act M_s. If N is a complement of X in M_s , then N⋂Y is a complement of X in Y.

Proof: From definition of complement and proposition (2.6) in (A.Shaymaa, 2017).■

Consider the following condition for an S-act M_s :

If X and Y are retracts of M_s , with X⋂Y closed in M_s , then X⋂Y is a retract of M_s(*).

Proposition(3.14) If M_s has (C_1^*), and satisfies the condition (*), then M_s is a GE-act.

Proof: Let Y$\dot{\cup}M_s$, and X be a closed subact of Y. It follows that X is closed in M_s . By (C_1^*), for M_s , there exists a complement N of X in M_s such that N is retract of M_s . By lemma (3.13), we have N⋂Y is a complement of X in Y; and hence a closed subact of Y. By the given condition (*), and since N is a retract of M_s , Y is a retract of M_s with N⋂Y is closed subact of M_s ; it follows that N⋂Y is a retract of M_s . This shows that any retract Y of M_s has (C_1^*) .Therefore M_s is a GE-act.■

Corollary (3.15): If GE-act M_s satisfies the condition (*), then M_s is extending.

Proof: It is obvious.■

4. Conclusion and Discussion

This research was motivated by the author's work where it was a limitation of 4 years from 2015 until now. From this research, we want to highlight on some important points which are:

1- Note that in the proof of theorem (3.2) we obtained a complement will be of the form$N_1 \dot{\cup} N_2$, where N_i is retract of M_i (i =1,2), for an arbitrary closed subact H of $M_s = M_1 \dot{\cup} M_2$. Thereby, as a direct consequence of this observation is the corollary (3.3).

This means that if M_i satisfies condition (C_1^*), then closed subact of M_s has a complement of the above form which implies that M_s is also satisfies (C_1^*).Besides, because of every subact of GE-act satisfies condition (C_1^*), the corollary (3.3) clarified that every retract of GE-act satisfies (C_1^*) (or has (C_1^*)).

2- We were remarkable that if M_s is an S-act with the property that X is Y-jective for every decomposition of $M_s = X \dot{\cup} Y$; then M_s need not have the condition (C_1^*).Actually, indecomposable acts need not to satisfy the condition (C_1^*).

3- Notice that the fact that essential extensions have the same complements in any acts M_s , allows us to replace subacts in the condition (C_1^*) by closed subacts.

4- Under condition (*), an S-act M_s which has (C_1^*) is equivalent to act with GE-act. This explains in proposition (3.14) where it is shown that any closed subact of an act M_s which is then will be retracted of M_s has C_1^*. Thereby M_s will be GE-act.

5- Under condition (*), GE-act is equivalent to an extending S-act M_s .This fact was demonstrated in corollary (3.15).It is obvious that any act to be extending act, it must satisfy the condition, every closed subact of act is a retract (or every subact of act is ⋂-large (essential) in retract (direct summand)) and GE-act needs the closed subact of it to be retract where the condition (*) complete this requirement.

For the future work, one can generalized this work to fully invariant extending act if each fully invariant subact of an S-act M_s is ⋂-large (essential) in a retract of M_s (or one can extend this work to Goldie extending act where an S-act M_s is called Goldie extending if for each subact A of M_s there exists a retract B of M_s such that A⋂B is ⋂-large (essential) in both A and B.

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