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UNCERTAINTY IN THE PERT'S CRITICAL PATH

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Abstract

In this paper, the problem of scheduling is addressed. Due to difficulties in scheduling projects, researchers and professionals have proposed a tremendous number of works aiming at finding the best method to accomplish this phase of any project, especially if the decision maker is facing the challenge of uncertain estimations. One of the most used families of techniques is discussed in this paper, namely the Fuzzy Program Evaluation and Review Technique techniques. This family of techniques is based mainly on using the classical Program Evaluation and Review Technique and the fuzzy set theory. This work presents a comparison between two interesting techniques used to tackle the problem of uncertainty, namely the Model for Project Scheduling with Fuzzy Precedence Links and the Centroid techniques. The first technique is based on the relationship strength between each two activities in order to resolve the problem of the critical path. The second technique is based on a very simple mathematical concept and arithmetic of fuzzy numbers to tackle the same *problem. Based on the results of a numerical example, we noticed that the simplicity and inexpensiveness of the Centroid method beat the complicated and expensive characteristics of the Model for Project Scheduling with Fuzzy Precedence Links.*

Keywords

Centroid Method, Fuzzy Set Theory, Fuzzy PERT, Model For Project Scheduling With Fuzzy Precedence Links

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1. Introduction

Generally, a project is a commitment based on an idea or need, to achieve a result or goal, which can be a product or a service. The implementation of this commitment requires human, financial and material resources, time (including a start and end dates) and usually an amount of money (Conchùir, 2010). In order to accomplish the project goals with an acceptable level of efficiency and effectiveness, it is necessary to focus on the project Management, where many processes especially the description, planning and scheduling, organization, monitoring and the control of resources dedicated to the project have to be well handled using, most of the time, specialized methods, techniques and tools. As mentioned above, planning and scheduling is among the most important processes in any project. We can distinguish two important network planning and scheduling techniques used to determine the critical path (a sequence of activities that do not tolerate any delay otherwise the whole project will not be finished at the expected date), the Critical Path Method (CPM) and the Program Evaluation and Review Technique (PERT) (Belharet, 2003; Fondahl, 1961; U.S. Dept. of the Navy. Program Evaluation Research Task, 1958). The first technique represents the project activities usually by an activity-on-node diagram is deterministic time estimation. On the other hand, project activities in the PERT technique are represented by an activity-on-arrow diagram, this technique allows integrating probabilities in order to estimate activities time and dates. As real-life projects are more complicated and characterized with uncertainty when it comes to activities' execution time estimation, it has been necessary of thinking of other methods that can take uncertainty into consideration. As mentioned before, PERT allows using probabilities in order to define the optimistic; most probable and pessimistic estimation for each activity in the project, yet this didn't give the needed flexibility in estimating the critical path. In order to overcome this impediment in estimating the critical path, some new techniques have been proposed in which the fuzzy set theory has been used to tackle the problem of uncertainty. In the next section we discuss the main idea of this theory.

2. Background

As we know, Program Evaluation and Review Technique (PERT) and Fuzzy set theory have become, respectively, almost the standard technique and theory that anyone thinks of when we start looking for a technique to schedule our projects and a theory to tackle the problem of uncertainty. Thanks to the concept of membership degree in the fuzzy sets theory (Bloch et al., 2003; L. Zadeh, 1965; L. A. Zadeh, 1975) which is completely different from the one known in the frame of the traditional set theory. In the fuzzy sets theory numbers could be represented as sets rather than values. The simplicity of representing a fuzzy number by just indicating its main three "points" (minimum, modal and maximum), makes of Triangular Fuzzy Numbers (TFN) a very handy tool to represent almost any uncertain estimation (see figure 01):

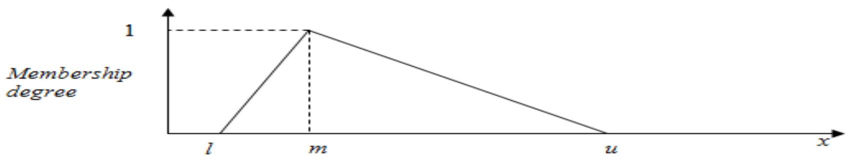

Figure 1: *Triangular Fuzzy Number* $A(l, m, u)$

If *A* is a Triangular Fuzzy Number, then its membership function $\mu_A(x)$ could be represented as follows,

$$\mu_A(x) = \begin{cases} 0 & x \leq l \\ \dfrac{x-l}{m-l} & l < x \leq m \\ \dfrac{u-x}{u-m} & m < x \leq u \\ 0 & x > u \end{cases} \quad (1)$$

Using this fuzzy representation, uncertain estimation of task's duration in the PERT network (Chanas & Kamburowski, 1981; Chanas & Zielinski, 2001; C.-T. Chen & Sue-Fen Huang, 2007; S.-P. Chen, 2007; Hsiau & R.Lin, 2009 ; Shankar & Saradhi, 2011; Yang, Chou, Lo, & Tseng, 2014), could be easily expressed.

In the next two sections, we give a brief presentation of two interesting methods, the *Centroid method* and *the Project Scheduling with Fuzzy Precedence Links*.

3. The Model For Project Scheduling With Fuzzy Precedence Links Method (MPSFPL)

Based on the TFN concept, the Fuzzy forward calculation and the backward fuzzy calculation, Mohammad Sharafi et al. (Sharafi, Jolai, Iranmanesh, & Hatefi, 2008) tried to tackle the problem of the critical path taking into account the relationship strength between each two activities.

3.1 Fuzzy Forward Calculation

The fuzzy earliest time $\widetilde{E}_j$ of occurrence of each event is calculated as follows,

$$\widetilde{E}_j = (e_j^a, e_j^b, e_j^c) = \begin{cases} \underbrace{M\tilde{a}x}_{i \in P(j)} \left\{ \widetilde{E}_i \oplus \widetilde{d_{ij}} \otimes \underbrace{M\tilde{a}x}_{k \in S(j)} \{\tilde{S}_{ijk}\} \right\} & \text{if } P(j) \neq \phi \\ \widetilde{T}_S = (t_S^a, t_S^b, t_S^c) & \text{if } P(j) = \phi \end{cases} \quad (2)$$

$$\widetilde{ES_{ij}} = \widetilde{E}_i = (es_{ij}^a, es_{ij}^b, es_{ij}^c) \quad (3)$$

$$\widetilde{EF_{ij}} = \widetilde{ES_{ij}} \oplus \widetilde{d_{ij}} \quad (4)$$

$$\widetilde{TF} = (tf^a, tf^b, tf^c) = \underbrace{M\tilde{a}x}_{i \in N} \widetilde{E}_i \quad (5)$$

Where,

$\widetilde{E}_j$: fuzzy earliest occurrence time of event j, $\tilde{S}_{ijk}$: the amount of dependence between activity (i,j) and activity (j,k)

$\widetilde{ES_{ij}}$: fuzzy earliest start of activity (I,j), $\widetilde{EF_{ij}}$: fuzzy finish start of activity (I,j)

$\widetilde{TF}$: fuzzy completion time of the project

3.2 Fuzzy Backward Calculation

We can calculate the latest fuzzy occurrence time $\widetilde{L}_i$ of each event as follows:

$$\widetilde{L}_i = \begin{cases} \underbrace{M\tilde{i}n}_{j \in S(i)} \left\{ \widetilde{L}_j - \widetilde{d_{ij}} \otimes \underbrace{M\tilde{a}x}_{k \in S(j)} \{\tilde{S}_{ijk}\} \right\} & \text{if } S(j) \neq \phi \\ \widetilde{T}_F & \text{if } S(j) = \phi \end{cases} \quad (6)$$

The latest fuzzy occurrence time L_i of event i:

$$L_i = (l_i^a, l_i^b, l_i^c) \quad (7)$$

$$S_{ijk} = (S_{ijk}^a, S_{ijk}^b, S_{ijk}^c) = \max \tilde{S}_{ijk} \quad (8)$$

$$l_i^c = \max\left(0, \underbrace{\min}_{j\in S(i)} \left(l_j^c - d_{ij}^c * S_{ijk}^c\right)\right) \quad (9)$$

$$l_i^b = \max\left(0, \min(l_j^c, \underbrace{\min}_{j\in S(i)} \left(l_j^b - d_{ij}^b * S_{ijk}^b\right)\right) \quad (10)$$

$$l_i^a = \max\left(0, \min(l_j^b, \underbrace{\min}_{j\in S(i)} \left(l_j^a - d_{ij}^a * S_{ijk}^a\right)\right) \quad (11)$$

The latest fuzzy finishing time $L\tilde{F}_{ij}$ of activity (i, j):

$$L\tilde{F}_{ij} = \tilde{L_j} = (lf_{ij}^a, lf_{ij}^b, lf_{ij}^c) \quad (12)$$

$$L\tilde{F}_{ij} = \tilde{L_j} - \widetilde{d_{ij}} \otimes \underbrace{M\tilde{a}x}_{k\in S(j)} \{\tilde{S}_{ijk}\} \oplus \widetilde{d_{ij}} \quad (13)$$

$$lf_{ij}^c = \max\left(0, \min(tf^c, \left(l_j^c - d_{ij}^c * S_{ijk}^c + d_{ij}^c\right)\right) \quad (14)$$

$$lf_{ij}^b = \max\left(0, \min(tf^b, lf_{ij}^c, \left(l_j^b - d_{ij}^b * S_{ijk}^b + d_{ij}^b\right)\right) \quad (15)$$

$$lf_{ij}^a = \max\left(0, \min(tf^a, lf_{ij}^b, \left(l_j^a - d_{ij}^a * S_{ijk}^a + d_{ij}^a\right)\right) \quad (16)$$

Fuzzy latest start time $L\tilde{S}_{ij}$ of activity (i, j):

$$L\tilde{S}_{ij} = (ls_{ij}^a, ls_{ij}^b, ls_{ij}^c) = \widetilde{LF_{ij}} - \widetilde{d_{ij}} \quad (17)$$

$$ls_{ij}^c = \max\left(0, \left(lf_{ij}^c - d_{ij}^c\right)\right) \quad (18)$$

$$ls_{ij}^b = \max\left(0, \min(ls_{ij}^c, \left(lf_{ij}^b - d_{ij}^b\right)\right)) \quad (19)$$

$$ls_{ij}^a = \max\left(0, \min(ls_{ij}^b, \left(lf_{ij}^a - d_{ij}^a\right)\right)) \quad (20)$$

3.3 Total Fuzzy Slack

The total fuzzy slack could be calculated as follows:

$$T\tilde{F}_{ij} = (tf_{ij}^a, tf_{ij}^b, tf_{ij}^c) = L\tilde{F}_{ij} - E\tilde{F}_{ij} \quad (21)$$

$$tf_{ij}^c = \max\left(0, \left(lf_{ij}^c - ef_{ij}^c\right)\right) \quad (22)$$

$$tf_{ij}^b = \max\left(0, \min(tf_{ij}^c, \left(lf_{ij}^b - ef_{ij}^b\right)\right)) \quad (23)$$

$$tf_{ij}^a = \max\left(0, \min(tf_{ij}^b, \left(lf_{ij}^a - ef_{ij}^a\right)\right)) \quad (24)$$

4. Centroid Method

Based on the concept of Triangular Fuzzy Number (TFN), The Centroid of a TFN *A* (Abbasbandy & Hajjari, 2011; Cheng, 1998) could be calculated as follows,

$$\mathrm{Centroid(A)} = \frac{l + m + u}{3} \quad (25)$$

Using fuzzy numbers' arithmetic operations (Gani, 2012; Gao, Zhang, & Cao, 2009), fuzzy forward and backward calculation could be calculated as explained in the previous method. Any activity (i, j) is taken as critical activity if centroid(i) ≤ 0 and centroid (j) ≤ 0.

5. Numerical Example

As a case study, we have applied the two aforementioned techniques on the problem tackled by Mohammad Sharafi et al. (Sharafi et al., 2008). Tables 01 and 02 gather the activities duration and relationship degrees respectively.

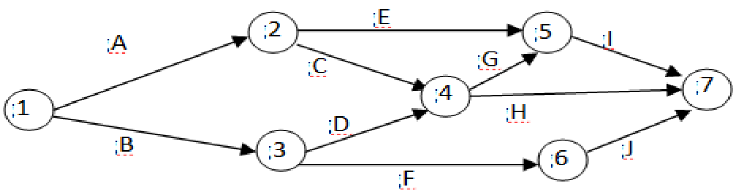

Figure 2: *Numerical example's PERT network*

Table 1: *The activities' duration*

Activity	I	T	Duration (m, l, u)		
A	1	2	25	28	32
B	1	3	40	55	65
C	2	4	32	37	43
D	3	4	20	25	35
E	2	5	35	38	42
F	3	6	42	45	55
G	4	5	20	25	28
H	4	7	60	65	75
I	5	7	65	75	85
J	6	7	15	18	22

Table 2: *Activities relationship's fuzzy degree*

Activities relationship	l	m	u
1-2-4	0.4	0.5	0.6
1-2-5	0.3	0.4	0.5
1-3-4	0.6	0.7	0.8
1-3-6	0.45	0.55	0.7
2-4-5	0.2	0.3	0.5
3-4-7	0.6	0.65	0.7
2-5-7	0.8	0.9	1
4-5-7	0.5	0.6	0.65

2-4-7	0.7	0.8	0.9
3-4-5	0.4	0.5	0.6
4-5-6	0.5	0.7	0.8
3-6-7	0.3	0.4	0.6

Based on the MPSFPL method the CP is 1-3-4-5-7.

For the Centroid method, the fuzzy earliest $\widetilde{L}_x$ and the latest $\widetilde{E}_x$ of events and the difference between them are represented in Table 03.

Table 3: *Events' Centroid*

Event	$\widetilde{E}_x$			$\widetilde{L}_x$			$\widetilde{L}_x - \widetilde{E}_x$			**Centroid** $(\widetilde{L}_x - \widetilde{E}_x)$
1	0	0	0	-68.7	0	68.7	-68.7	0	68.7	0
2	10	14	19.2	-30.9	25.15	82.3	-50.1	11.15	72.3	11.117
3	24	38.5	52	-16.7	38.5	92.7	-68.7	0	68.7	0
4	36	54.75	76.5	7.8	54.75	104.7	-68.7	0	68.7	0
5	46	69.75	94.7	26	69.75	114.7	-68.7	0	68.7	0
6	30.6	48.25	78.5	89	126.75	164.7	10.5	78.5	134.1	74.367
7	111	144.75	179.7	111	144.75	179.7	-68.7	0	68.7	0

Using the Centroid method, the CP is 1-3-4-5-7.

As we can notice, the two methods have yielded the same CP, yet the important number of instructions (which means execution time) needed to get this CP using MPSFPL makes us tend to consider that the Centroid method is much more advantageous, because of its efficiency and remarkable simplicity.

6. Conclusion

There are two important points we could conclude this paper with. First, we have seen that the fuzzy sets theory still a very good choice when we need to represent uncertainty, either to give a comprehensible representation for estimations, or to combine several fuzzy estimations in order to get new insights about our projects. The second point is about the comparison we have made in the previous section. Usually, complicated methods yield better results despite their cumbersome concepts and implementation, but surprisingly, in the case study, we found that the simplest method, namely Centroid method, had overcome the complicated one (MPSFPL) and gave the same results with less code!

6.1 Research Limitations

Despite the fact that, in numerical example we demonstrated above, the centroid method has shown promising performances against another complicated method (MPSFPL), we can't conclude that this simple method could stand if we apply it in a large-scale project, especially if we use it in a real one.

6.2 Scope Of Future Researches

In our future works, we aim at improving the Centroid method and adapting it to the context of dynamic scheduling, where the challenge of updating the CP in a large-scale real project, will be in the spotlight.

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