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DIFFERENCE IN SURFACE FITTING WITH STANDARD AND MATRIX TOPOLOGY

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Abstract

This paper presents the impact of the surface topology of the scanned 3D object on parametric fitting. Whether it is a simple NURBS (Non-uniform rational B-spline) or a more complex hierarchical spline version, it is important to apply the fitting procedure. Here we describe the differences between fitting a surface with a given topology as a result of a 3D scanning system and a matrix topology of the surface, where the original surface is replaced by the result of a preset *number of sections of the original geometry. We use the matrix and the free-form distribution. The former is more stable with respect to the distribution of the control point, the latter is numerically more suitable. In the future, we plan to adopt the free-form distribution to utilize the advantages of both distributions.*

Keywords

Topology, NURBS, Hierarchical Spline, Surface Fitting

1. Introduction

In computer-aided design (CAD) and artificial intelligence (AI), the referent model mainly comes from the 3D scanning process, which results in a triangulated 3D point cloud. This type of model is not suitable for re/modelling, optimization and shape synthesis. The main task required by CAD (Li et al., 2013; Marinić-Kragić et al., 2016; Su et al., 2017) or AI (Siqueira et al., 2020) is presenting/fitting the same model with one of a mathematical parameterization model such as NURBS, T-spline, THB-spline (truncated hierarchical B-spline), which faithfully represents the geometry of the 3D point cloud model and has an embedded smoothness that can be intuitively modified by itself. In such a fitting process, we are confronted with challenges related to the topology of the input model.

In this paper we describe the implications for fitting a parametric model to a scanned 3D model with different surface topologies. A brief introduction to the creation of a parametric NURBS model is given in this paper, while detailed information can be found in (Ćurković et al., 2017; Ćurković et al., 2018; Ćurković & Vučina, 2014). The work shows that the fit of the parametric model in the case of a free-form surface topology (original triangulation) depends on the density of the triangulation mesh and, also on the distribution of vertices in the surface mesh. As a result, the control points of the parametric model are grouped in areas with a higher density of grid points. In the case of a uniform distribution of vertices in the surface mesh, a uniform distribution of control points of the parametric model is to be expected. Using the matrix distribution of the vertices of the model surface in the fitting process leads to a more uniform distribution of the control points of the parametric model, independent of the density of the triangulated mesh. In our applications, we still use both distributions, but the goal is to avoid the matrix distribution, as additional effort is required to achieve a uniform distribution of the control points of the parametric model, and to improve the free-form distribution, which gives the same quality as the matrix distribution.

2. Comparison of Fitting with Different Topologies

In the following, we outline the definition of the standard NURBS parametric model, which is sufficient to demonstrate the effects of different surface topologies when fitting the parametric model to the given model surface. The NURBS surface is defined by its control points, weight factors, degrees of polynomials and the set of knots,

$$\boldsymbol{C}(u,v)=\frac{\sum_{i_0=0}^{n_0}\sum_{i_1=0}^{n_1}N_{i_0,p_u}(u)N_{i_1,p_v}(v)w_{i_0i_1}\boldsymbol{Q}_{i_0i_1}}{\sum_{i_0=0}^{n_0}\sum_{i_1=0}^{n_1}N_{i_0,p_u}(u)N_{i_1,p_v}(v)w_{i_0i_1}}\in\mathbb{R}^3 \qquad (1)$$

where n_0, n_1 are the numbers of the control points, and $N_{i_0,p_u}(u)$ and $N_{i_1,p_v}(v)$ are the basic B-spline functions of degrees $p_u, p_v \in \mathbb{N}$ defined by

$$N_{i,0}(u)=\begin{cases}1, & \bar{u}_i \le u \le \bar{u}_{i+1}\\ 0, & else\end{cases},$$
$$N_{i,p}(u)=\frac{u-\bar{u}_i}{\bar{u}_{i+p}-\bar{u}_i}N_{i,p-1}(u)+\frac{\bar{u}_{i+p+1}-u}{\bar{u}_{i+p+1}-\bar{u}_{i+1}}N_{i+1,p-1}(u). \qquad (2)$$

The knots $\bar{u}_i \in [0,1]$, as part of the basic B-spline functions, we set as

$$\bar{\boldsymbol{u}}=\left\{\underbrace{0,\dots 0}_{p+1},\bar{u}_{p+1},\dots,\bar{u}_n,\underbrace{1,\dots 1}_{p+1}\right\},\ \left\{\bar{u}_i=\frac{i}{n}\right\}_{i=p+1}^{n}. \qquad (3)$$

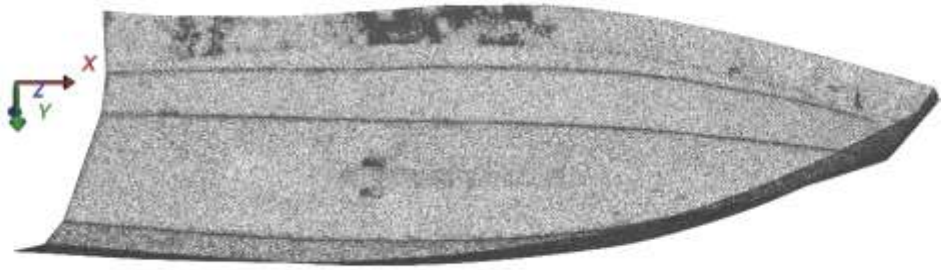
Matrix $\boldsymbol{Q}$ presents control points.

$$\boldsymbol{Q}=\begin{bmatrix}\boldsymbol{Q}_{00} & \cdots & \boldsymbol{Q}_{0n_1}\\ \vdots & \ddots & \vdots\\ \boldsymbol{Q}_{n_00} & \cdots & \boldsymbol{Q}_{n_0n_1}\end{bmatrix}\in\mathbb{R}^{3(n_0+1)\times(n_1+1)}. \qquad (4)$$

Figure 1 shows an example of a model with the original surface density provided by the scanner system. It is the first example to which we fitted NURBS parametric model using two different surface topologies.

Figure 1: *Example of a Model with the Original Surface Density Provided by the Scanner System*

a) Shaded surface.



b) Triangulation of a)

(Source: Self/Authors' Own Illustration)

The following equation shows the error function when fitting the NURBS model to the surface with the original surface topology, where $\boldsymbol{P_j}$ represents the vertices of the triangulated mesh.

$$\boldsymbol{E}_{freeform}(\boldsymbol{Q}) = \frac{1}{2}\sum_{j=0}^{m} \left\| C(u_j, v_j) - \boldsymbol{P}_j \right\|^{\mathbf{2}} \quad (5)$$

The next equation

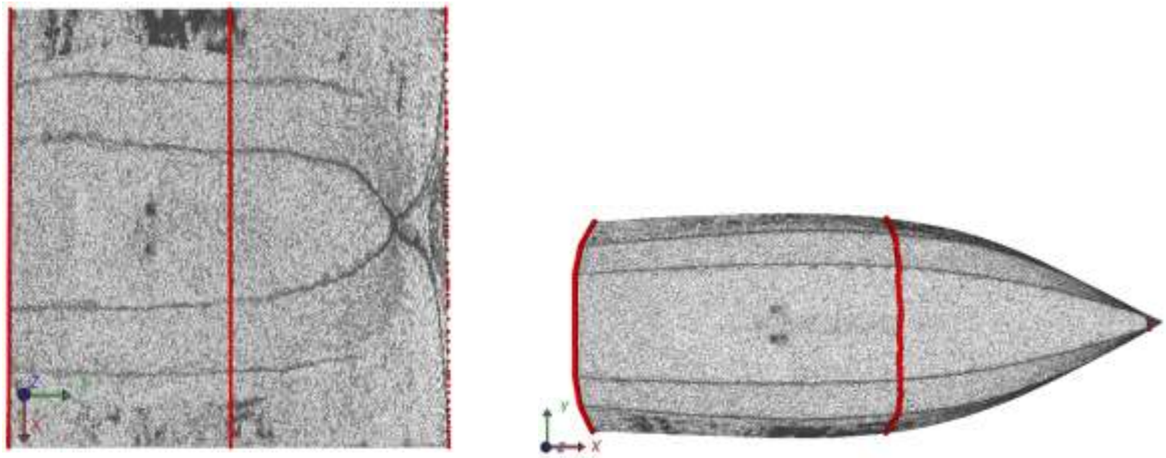
$$\boldsymbol{E}_{matrix}(\boldsymbol{Q}) = \frac{1}{2}\sum_{j_0=0}^{m_0}\sum_{j_1=0}^{m_{\mathbf{1}}} \left\| C(u_{j_0 j_1}, v_{j_0 j_1}) - \boldsymbol{P}_{j_0 j_1} \right\|^{\mathbf{2}} \quad (6)$$

assumes that the model surface is represented in matrix form,

$$\boldsymbol{P} = \begin{bmatrix} \boldsymbol{P}_{00} & \cdots & \boldsymbol{P}_{0m_1} \\ \vdots & \ddots & \vdots \\ \boldsymbol{P}_{m_0 0} & \cdots & \boldsymbol{P}_{m_0 m_1} \end{bmatrix} \in \mathbb{R}^{3(m_0+1)\times(m_1+1)} \quad (7)$$

where m_0 is the number of sections and m_1 is the number of vertices in each section (see Figure 2).

Figure 2: *The Approach to Obtain the Matrix Topology of the Given Surface*



a) Sections of the Surface in the Parametric Domain

b) Corresponding Points from a) On the Original Surface

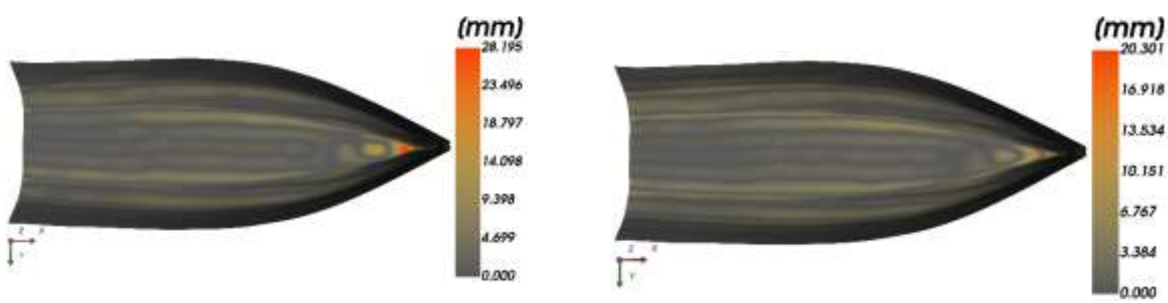
(Source: Self/Authors' Own Illustration)

A detailed description of the matrix form of the surface can be found in (Ćurković et al., 2017, 2018). The next figure below shows the result of fitting NURBS to the model with the original free form surface topology and matrix topology.

Figure 3: *The Result of NURBS Fitting to the Model with the Original Surface Topology and Matrix Topology*

a) NURBS with Free Form Surface Topology

b) NURBS with Matrix Form Surface Topology



c) Distribution of the Distance Between a) and the Original Surface Form Figure 1

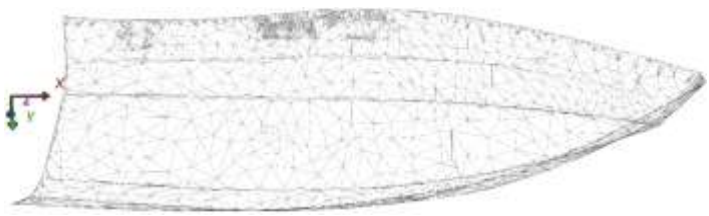
d) Distribution of Distance between b) and the Original Surface Form Figure 1

(Source: Self/Authors' Own Illustration)

The above example shows a small difference in the distribution of control points between two NURB models (Figure 3a and Figure 3b), which leads to a small difference in the error distribution (Figure 3c and Figure 3.d).

The larger difference in the distribution of control points and the corresponding geometry differences arise in the case of a thinner surface triangulation grid, as shown in Figure 4. This kind of change affects the connection of multiple NURBS surfaces into a more complex hierarchical spline (Giannelli et al., 2016; Hong Qin, 1995), the creation of models of displacement surfaces (Lee et al., 2000), and so on.

Figure 4: *The Result of NURBS Fitting to the Thinned Model with the Original Surface Topology and Matrix Topology*



a) The Thinned Surface of the Surface from Figure 1

b) NURBS with Free Form Surface Topology

c) NURBS with Matrix Form Surface Topology

d) Distribution of the Distance between b) and the Original Surface Form a)

e) Distribution of Distance between c) and the Original Surface Form a).

(Source: Self/Authors' Own Illustration)

3. Conclusion

As we expected, the matrix distribution mainly leads to more stable solutions concerning the distribution of the control points of the parametric model. On the other hand, the free-form distribution leads to a numerically faster solution. In the future, we plan to adopt our method of projection into a parametric rectangular domain based on harmonic mapping, so that the redistribution of vertices in 2D depends on whether the vertices belong to geometric features or not. In this way, we can avoid the need for the matrix form of geometry.

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